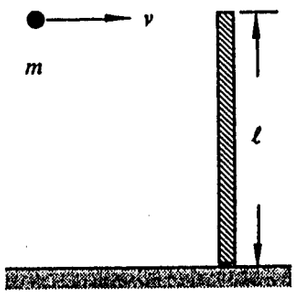
1.(16%) A thin rod of mass M, length ℓ, and constant density is standing on end on a rough table that forms the xy-planes. A putty ball of mass $m = M/3$ is thrown with velocity $\mathbf{v} = v\,\hat{i}$ and hits the top of the thin rod. If the putty ball makes a completely inelastic collision, and if the point of contact between the rod and the table does not move.

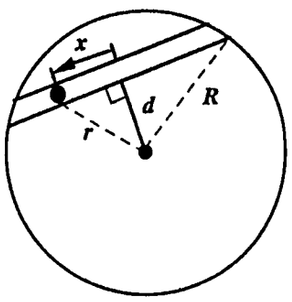


(a) Find the moment of inertial of the compound system(i.e. after the ball struck).

(b) What is the angular velocity of the compound system as it hits the table?

(c) Find the angular momentum and the kinetic energy as it hits the table.

2.(16%) Consider a tunnel drilled along a chord of the earth, as shown in the diagram.

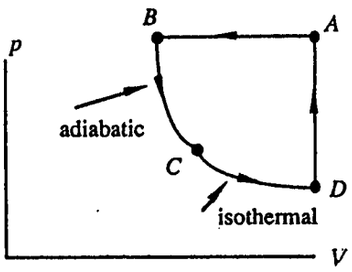


(a) Find the potential energy of a mass m placed in such a tunnel as a function of x.

(b) What is the period of the motion if the ball is released at rest at an entrance to the tunnel?

$R = 6.374 \times 10^6$ m

3.(20%) A system of dilute helium gas is taken through the following reversible cycle on a p-V diagram: AB is an isobaric compression; BC is an adiabatic expansion; CD is an isothermal expansion; DA is an isochoric process. Find the heat flow, work done on the system, change in temperature, and change in entropy for each leg of the cycle in terms of the quantities T_A, T_B, T_C, and p_A.



4.(16%) Two very large flat plates are parallel and separated by a distance D. The side of the left plate that faces the right side plate has a surface charge density $+\sigma$. The inside surface of the right side plate has a surface charge density $-\sigma$.

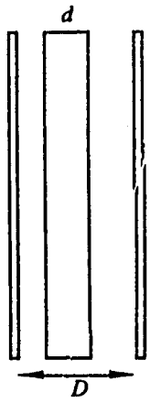
(a) What are the electric fields between and outside the plates?

(b) The potential difference?

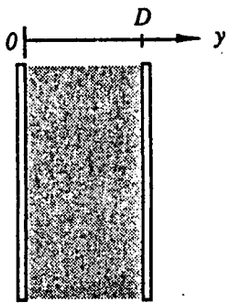
Now another uncharged metal plate of thickness d is inserted, if the charge density of the original plates is unaltered.

(c) What are the electric fields in the gaps?

(d) The potential difference between the two outer plates?



5.(16%) A parallel-plate capacitor has an area of $L \times L$ and a plate separation of $D << L$. It is filled with a non-uniform dielectric whose dielectric constant varies linearly from one plate to another. At the bottom plate, the dielectric constant is κ_0, whereas at the upper plate, it is κ_1. If y is the distance measured up from the bottom plate, then $\kappa = \kappa_0 + [(\kappa_1 - \kappa_0)y/D]$. Calculate the capacitance of this capacitor.



6.(16%) A metal ring is constructed so as to expand or contract freely. In a region with a constant magnetic field B_0 oriented perpendicular to it, the ring expands, with its radius growing linearly with time as $r = r_0 (1 + \alpha t)$. As the ring expands and grows thinner, its resistance per unit length changes according to the empirical rule $R = R_0 (1 + \beta t)$. Find the current induced in the ring as a function of time. Please specify the direction as well as the magnitude of the current.