

- (1) Let $\{x_n\}$ be an arbitrary real sequence.
- (i) Suppose that $\lim_{n \rightarrow \infty} x_n = r$, what is $\lim_{n \rightarrow \infty} |x_n| = ?$ 5%
 - (ii) Conversely, if $\lim_{n \rightarrow \infty} |x_n| = r$, what is $\lim_{n \rightarrow \infty} x_n = ?$ 5%
- (2) Assume $f''(x) = (x+4)e^{\frac{x-1}{2}}$, $f'(1) = -10$. Please find
- (i) where f is concave up and concave down? 5%
 - (ii) where f is increasing and decreasing? 5%
- (3) Let $z = f(x, y) = \sqrt{16 - x^2 - y^2}$.
- (i) Find the domain and the range of f . 5%
 - (ii) Sketch the level curves of f for $z = 0, 1, 2, 3, 4$. 5%
 - (iii) In what direction does f decrease most rapidly at the point $(-1, \sqrt{3})$? 5%
 - (iv) Find all the critical points of f . 5%
- (4) Let $f(x) = e^x + e^{-x} + 2 \cos x$.
- (i) Compute the third order Taylor's expansion with remainder around $x = 0$. 10%
 - (ii) Use (i) to show that $f(x)$ has a local minimum at $x = 0$. 10%
- (5) If f is a continuous function, find the value of the integral 10%

$$I = \int_0^a \frac{f(x)}{f(x) + f(a-x)} dx.$$

(Hint: let $u = a - x$ and use the substitution method.)

- (6) Compute the double integral of $f(x, y) = \sqrt{|y - x^2|}$ over $S = [-1, 1] \times [0, 2]$. 10%
(Hint: divide S into two regions $\{(x, y) \mid y < x^2\}$ and $\{(x, y) \mid y \geq x^2\}$)
- (7) The Ross-Simons Company has a monthly advertising budget of \$60,000. Their marketing department estimates that if they spend x dollars in newspaper advertising and y dollars on television advertising, then the monthly sales will be given by $z = f(x, y) = 90x^{1/4}y^{3/4}$ dollars.
- (i) Determine how much money Ross Simons should spend on newspaper advertising and on television advertising per month in order to maximize its monthly sales. 10%
 - (ii) If the company decides to throw in additional \$2,000 as the advertising budget, use the Lagrange multiplier to estimate what the maximum monthly sales will be under the new budget plan? 10%