(1) The *Lorenz* function $L(x,t)$ gives, after t years of World War II, the proportion of total income earned by the lowest proportion x of the population. For example, $L(0.5,30)=0.25$ means, 30 years after War, those persons, whose income ranks among the bottom half of the population, earn only a quarter of the total social wealth.

(i) What is the *Lorenz* function for a society in which everyone earns exactly the same income? What is the *Lorenz* function for a society in which the total wealth belongs to a single dictator? 10%

(ii) Define the *Gini index* as 10%

$$G(t)=2\int_0^1 [x-L(x,t)]dx.$$

If $G(t)$ is a decreasing function, what does it imply?

(2) The following table shows the cost of one year of tuition with room and board at a private four year college. Find an exponential curve $y=Be^{Ax}$ to fit the data so that the squared deviations for the logarithms of the cost is minimized. (Note: $e^{2.448}=11.53$) 10%

	x	y =cost	$\ln y$
1995-6	1	12.2	2.5
1996-7	2	13.0	2.56
1997-8	3	13.8	2.62
1998-9	4	14.5	2.67

(3) Let $z=f(x,y)=x^2-y^2+3$ and $g(x,y)=2x+y-3$.

(i) Find (x^*,y^*) that maximizes $f(x,y)$ subject to $g(x,y)=0$ using Lagrange multipliers method. What is the Lagrange multiplier at (x^*,y^*). 10%

(ii) Sketch the level sets of f for $z=3,4,5,6,7$ and $g(x,y)=0$. Plot the constrained minimum point (x^*,y^*); $\nabla f(x^*,y^*)$ and $\nabla g(x^*,y^*)$ on this graph. 10%

(iii) Interpret the geometrical meaning of Lagrange multipliers using the graph. 10%

(iv) Determine the *minimum* points of f subject to $-g(x,y)\le 0$ by modifying and inspecting the above graph. 10%

(4) The emergency stopping distance S in feet for a truck of weight w tons traveling at v miles per hour on a dry road is 10%

$$S=\frac{0.01w^2v^3}{\sqrt{w^2+v^2}}.$$

For a gravel truck that weights 30 tons and is usually driven at 40 miles per hour, if now it has extra 10 tons of load, use total differential to estimate how much slower the truck driver has to do in order to keep the same emergency stopping distance?

(5) Compute $\int\int x^2(\ln tx)^2 dxdt$. 10%

(6) Let $f(x)=(1+x)^r$ where $r\in\mathbb{R}-\{0\}$.

(i) Find Taylor series (with radius of convergence) around $x=0$ if r is a positive integer. 5%

(ii) Find Taylor series (with radius of convergence) around $x=0$ if r is NOT a positive integer. 5%