

20% 1. Find the similarity transformation matrix that diagonalizes matrix  $A$ , where

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

20% 2. Derive with clear steps the Laplace transform for the following two functions.

a.  $\cos^2 t$

b.  $\frac{2}{t}(1 - \cos \omega t)$

20% 3. Solve the ordinary differential equation of

$$(x^2 + 1) \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} + 3x^{-2} = 0$$

20% 4. For a given curve

$$\mathbf{r}(t) = 3\cos t \mathbf{i} + 3\sin t \mathbf{j} + 4t \mathbf{k}$$

find the following

- the unit tangent vector  $\mathbf{u}(t)$ ,
- the tangent at  $\mathbf{P} : (0, 3, 2\pi)$ , and
- the arc length from  $\mathbf{P} : (0, 3, 2\pi)$  to  $\mathbf{Q} : (-3, 0, 4\pi)$ .

20% 5. Let  $X$  have the density function  $f(x) = k(x - 0.9)(1.1 - x)$  if  $0.9 < x < 1.1$  and 0 otherwise. Determine constant  $k$ , mean  $\mu$  and variance  $\sigma^2$ .