93學年度國立成功大學研究所招生考試	交通管理科學系 丙組 微積分 J組	試題	共 1 頁 第 1 頁

8 points (1) Find the constant c and d such that the following function is continuous:

$$f(x)=\begin{cases}4 & \text{if } x\le -1,\\ cx+d & -1<x<3,\\ -4 & \text{if } x\ge 3.\end{cases}$$

10 points (2) Find an equation of the tangent line to the graph of the function

$$3x^3y^2-3xy-y^3+1=0$$

at $(1,1)$.

18 points (3) Find the integrals

(a) $\int_0^4 |2x-1|\,dx$, (b) $\int (x+2)\sqrt{x-3}\,dx$, (c) $\int_0^2 \frac{1}{\sqrt[3]{x-1}}\,dx$.

8 points (4) Find the volume of the solid bounded by the graphs of $z=(xy)^2$, $z=0$, $y=0$, $y=4$, $x=0$, and $x=4$.

16 points (5) Find the volume of the solid obtained by revolving the curve $y=x^2/2$ from $(0,0)$ to $(1,1/2)$

(a) about the line $y=0$ (the x-axis), and

(b) about the line $x=-1/2$.

16 points (6) A manufacturer's production is modeled by the Cobb-Douglas function

$$f(x,y)=100x^{3/4}y^{1/4}$$

where x represents the units of labor and y represents the units of capital. Each labor unit costs \$150 and each capital unit costs \$250. the total expenses for labor and capital cannot exceed \$50000.

(a) Find the maximum production level.

(b) Suppose that \$20000 more is available for labor and capital. What is the maximum number of units that can be produced?

8 points (7) Let (x_0, p_0) be the point at which a demand function and a supply function intersect. Economists call the area of the region bounded by the graph of the demand function, the horizontal line $y=p_0$, and the vertical line $x=0$ the *consumer surplus*. Similarly, the area of the region bounded by the graph of the supply function, the horizontal line $p=p_0$, and the vertical line $x=0$ is called the *producer surplus*.

Now, suppose that the demand function for a product is given by $p=-0.1x+10$ and the supply function for the same product is given by $p=0.3x+2$, where x is the number of units (in millions). Find the consumer and producer surpluses for this product.

16 points (8) A company manufactures a product at two locations. The costs of manufacturing x_1 units at plant 1 and x_2 units at plant 2 are

$$C_1=0.03x_1^2+4x_1+300$$

and

$$C_2=0.05x_2^2+7x_2+175,$$

respectively. If the product sells for \$10 per unit, find x_1 and x_2 such that the profit is maximized.

9 MS09 三-2