20% 1. If there is some positive integer m such that $\mathbf{A}^m = 0$, $\mathbf{A}$ is said to be **nilpotent**.

    a. Show that a nilpotent matrix is singular.

    b. If $\mathbf{A}$ is nilpotent, show that

$$(\mathbf{I} - \mathbf{A})^{-1} = \mathbf{I} + \mathbf{A} + \mathbf{A}^2 + \cdots + \mathbf{A}^{m-1}$$

20% 2. Assume that the position of a small fish can be defined by the Cartesian coordinates as $x = 6t$, $y = t + 2$, and $z = e^{-t}$, where t is the time, and the temperature distribution in the water is given by

$$T(x, y, z, t) = \left(5 + \frac{2}{x^2 + y^2 + t} \right) e^{-z}.$$

Determine the time rate of change of temperature experienced by the fish when $t = 0$.

20% 3. Solve the following ordinary differential equation with initial conditions

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = 2\cos t - sin 2t, \quad y(0) = 1, \quad \text{and} \quad \frac{dy}{dt}(0) = 0.5.$$

20% 4. Express the following values in complex $a + bi$ form

    a. $log(-2)$　　　　　b. $(2i)^{3/2}$

20% 5. Find the Fourier series of the function

$$f(x) = \begin{cases} 0 & \text{if} \quad -2 < x < -1 \\ k & \text{if} \quad -1 < x < 1 \\ 0 & \text{if} 1 < x < 2 \end{cases}$$