(1) (10 points) A business has a cost (in dollars) of $C = 0.5x + 500$ for producing x units. Find the limit of the average cost per unit as x approaches infinity.

(2) (10 points) Where, if anywhere, is the graph of $y = \sqrt{x}$ parallel to the line $\frac{1}{8}x - 8y = 1$.

(3) (10 points) A point is moving along the graph of $y = x^2$ so that dx/dt is 2 centimeters per minute. Find dy/dt for $x = 3$.

(4) (10 points) A Middle Eastern oil-producing country estimates that the demand for its oil (in millions of barrels) is $D(p) = 28e^{-0.04p}$, where p is the price of a barrel of oil. To raise its revenues, should it raise or lower its price from its current level of \$20 per barrel?

(5) (10 points) Find a function $f(x, y)$ such that $f_x(x, y) = 2xy$, $f_y(x, y) = x^2 + 2$, and $f(0, 1) = 4$.

(6) (10 points) A company manufactures two products, in quantities x and y. Because of limited materials and capital, the quantities produced must satisfy the equation $2x^2 + 5y^2 = 32500$. If the company's profit function is $P = 4x + 5y$ dollars, how many of each product should be made to maximize profit? Also find the maximum profit.

(7) (10 points) A company's marginal cost function is $MC(x) = xe^{-x/2}$ and fixed costs are 200. Find the cost function.

(8) (10 points) Economists use *Lorenz curves* to illustrate the distribution of income in a country. Letting x represent the percent of families in a country and y the percent of total income, the model $y = x$ would represent a country in which each family had the same income. The Lorenz curve, $y = f(x)$, represents the actual income distribution. The area between these two models, for $0 \le x \le 100\%$, indicates the "income inequality" of a country. In 2000, the Lorenz curve for the United States could be modeled by

$$y = x^{2.4}, \qquad 0 \le x \le 100\%$$

where x is measured from the poorest to the wealthiest families. Find the income inequality for the United States in 2000.

(9) (10 points) Evaluate $\displaystyle\int_0^1 \int_{\sqrt{y}}^1 \sqrt{x^3 + 1}\, dx\, dy$.

(10) (10 points) Find the sum of the infinite series $\displaystyle\sum_{k=0}^{\infty} \frac{(\ln 2)^k}{k!}$.