20% 1. For the equation given as

$$a \cosh x + b \sinh x = c, \qquad c > 0, .$$

    a. find the conditions with a, b, and c under which has zero, one, or two real solutions for x.

    b. what is the solution for x if $a^2 = c^2 + b^2$?

20% 2. Determine the work done in moving a particle in the force field
$\mathbf{F} = 3x^2\mathbf{i} + (2xz - y)\mathbf{j} + z\mathbf{k}$.

    a. along the curve defined by $x^2 = 4y$, $3x^3 = 8z$ from $x = 0$ to $x = 2$.

    b. Is $\mathbf{F}$ a conservative force? Explain.

20% 3. Consider a set of ordinary differential equations

$$\frac{dx}{dt} + 4\frac{dy}{dt} + x = 1,$$
$$\frac{dy}{dt} + \frac{dx}{dt} + y = t,$$

which subjects to $x = 1$ and $y = 0$ at $t = 0$. Solve $x(t)$ and $y(t)$ for $t \geq 0$.

20% 4. Consider the quadratic form

$$Q = x_1^2 + 4x_1x_2 - 4x_1x_3 - 6x_2x_3 + \alpha(x_2^2 + x_3^2).$$

    a. Express it in the form $\mathbf{x}^{\mathbf{T}}\mathbf{Ax}$, where $\mathbf{x} = [x_1 \ x_2 \ x_3]^T$ and $\mathbf{A}$ is a symmetric matrix.

    b. Determine the range of values of α for which Q is positive definite.

    c. For $\alpha = 1$, $\mathbf{A}$ is positive, positive-semi, negative, negative-semidefinite, or none of above.

20% 5. Find the real and imaginary parts of the functions

    (a) $e^z \sinh z$　　　　　(b) $\cos 2z$

for $z = x + iy$ and $x, y \in R$.