本試題是否可以使用計算機：　□可使用　，　☑不可使用　（請命題老師勾選）

1. TRUE or FALSE:
   (a) [5 points] If f is differentiable at $(0,0)$, then f is continuous at $(0,0)$.
   (b) [5 points] If f is a continuous function defined on the region $x^2 + y^2 \le 9$, then f has a maximum value and a minimum value in this region.
   (c) [5 points] If $f_x(0,0)$ exists, and $f_y(0,0)$ exists, then f is differentiable at $(0,0)$.
   (d) [5 points] If f is differentiable at $(0,0)$, then the tangent plane to the graph of f at $(0,0)$ is given by $z = f(0,0) + f_x(0,0)x + f_y(0,0)y$.

2. Let the following function be defined for all points (x,y) in the plane.
$$f(x,y) = 2xy - x^4 - x^2 - y^2$$
   (a) [5 points] Write the gradient of the function f.
   (b) [5 points] Write the Hessian matrix of f.
   (c) [5 points] Use the gradient to find a local extremum of f. Identify this extremum as a minimum, a maximum or neither.

3. Consider the function $f(x) = xe^{-x}$. Note that $\lim\limits_{x \to \infty} xe^{-x} = 0$. In this question, you need to find the first and second derivatives of the function f and to find all important features of its graph. Please write down your final answers neatly and note that a correct answer to some questions may be "none".
   (a) [5 points] horizontal asymptotes:
   (b) [5 points] $\lim\limits_{x \to -\infty} xe^{-x}$
   (c) [5 points] local extrema (with (x,y) coordinates and type – max or min):
   (d) [5 points] interval(s) where the function is concave up:
   (e) [5 points] inflection point(s) (with (x,y) coordinates):

4. [10 points] Find $\displaystyle\int \frac{\sqrt{3 + 2\ln x}}{x}\,dx$

5. [10 points] Solve the differential equation and initial condition $\begin{cases} y' = xy - x \\ y(0) = 4 \end{cases}$

6. [10 points] Find $F''(3)$ if $F(x) = \displaystyle\int_2^x \cos(\ln t)\,dt$

7. [10 points] Let $y = x^{\arcsin(x)}$. Find $\dfrac{dy}{dx}$.