編號： 405　系所：交通管理科學系乙組　　　　科目：作業研究

本試題是否可以使用計算機：　☑可使用 ，　□不可使用　（請命題老師勾選）

1. WSP Publishing sells textbooks to college students. WSP has two sales representatives available to assign to the A-G state area. The number of college students (in thousands) in each state is given in Figure 1. Each sales representative must be assigned to two adjacent states. For example, a sales representative could be assigned to A and B, but not A and D. WSP's goal is to maximize the number of total students in the states assigned to the sales representatives. Formulate an IP whose solution will tell you where to assign the sales representative. Do not solve your IP. (20%)
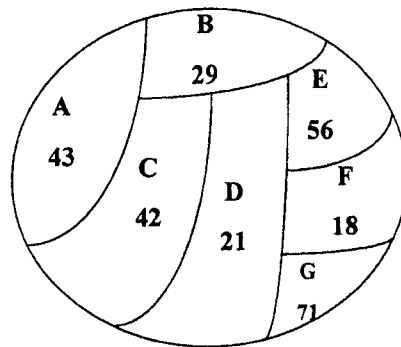


Figure 1.

2. The State College admissions office has modeled the path of a student through State College as Markov chain:

|  | F. | So. | J. | Sen. | Q. | G. |
|---|---|---|---|---|---|---|
| Freshman | .1 | .80 | 0 | 0 | .10 | 0 |
| Sophmore | 0 | .10 | .85 | 0 | .05 | 0 |
| Junior | 0 | 0 | .15 | .80 | .05 | 0 |
| Senior | 0 | 0 | 0 | .10 | .05 | .85 |
| Quits | 0 | 0 | 0 | 0 | 1 | 0 |
| Graduates | 0 | 0 | 0 | 0 | 0 | 1 |

Each student's state is observed at the beginning of each fall semester. For example, if a student is a junior at the beginning of the current fall semester, there is an 80% chance that he will be a senior at the beginning of the next fall semester, a 15% chance that he will still be a junior, and a 5% chance that he will have quit. (We assume that once a student quits, he/she never reenrolls.

(a) If a student enters State College as a freshman, how many years can he/she expects to spend as a student at State? (10%)

(b) What is the probability that a freshman graduates? (10%)

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3. Consider the following LP and its optimal tableau (Table 1).

$$Max \quad z = c_1 X_1 + c_2 X_2$$
$$s.t. \qquad a_{11}X_1 + a_{12}X_2 \le b_1$$
$$\qquad a_{21}X_1 + a_{22}X_2 \le b_2$$
$$\qquad X_1, X_2 \ge 0$$

Table 1.

| z | $X_1$ | $X_2$ | $S_1$ | $S_2$ | b |
|---|-------|-------|-------|-------|---|
| 1 | 0 | 0 | 2 | 3 | 2.5 |
| 0 | 1 | 0 | 3 | 2 | 2.5 |
| 0 | 0 | 1 | 1 | 1 | 1 |

Determine  $c_1, c_2, b_1, b_2, a_{11}, a_{12}, a_{21}$, and $a_{22}$. (20%)

4. Consider the following LP:

$$Max \quad z = 20X_1 + 10X_2$$
$$s.t. \qquad X_1 + X_2 = 150$$
$$\qquad X_1 \qquad \le 40$$
$$\qquad \qquad X_2 \ge 20$$
$$\qquad X_1, X_2 \ge 0$$

Find the dual to this LP. (20%)

5. Telephone calls from New York to Los Angeles are transported as follows: The call is sent first to either Chicago or Memphis, then routed through either Denver or Dallas, and final sent to Los Angeles. The number of phone lines joining each pair of cities is shown in Table 2.

Table 2.

| Cities | No. of Telephone Lines |
|--------|------------------------|
| N.Y. – Chicago | 500 |
| N.Y. – Memphis | 400 |
| Chicago – Denver | 300 |
| Chicago – Dallas | 250 |
| Memphis – Denver | 200 |
| Memphis – Dallas | 150 |
| Denver – L.A. | 400 |
| Dallas – L.A. | 350 |

Formulate an LP that can be used to determine the maximum number of calls that can be sent from New York to Los Angeles at any given time. (20%)