

本試題是否可以使用計算機:  可使用,  不可使用 (請命題老師勾選)

1. Evaluate the following integrations.

(a)  $\int_1^2 x^3 \ln x dx$ . 5%

(b)  $\int_0^1 \int_0^{\sqrt{1-x^2}} \sin(x^2 + y^2) dy dx$ . 5%

2. Find the following limits.

(a)  $\lim_{\alpha \rightarrow 0} \frac{\alpha}{\sqrt{1-\cos \alpha}}$ . 5%

(b)  $\lim_{n \rightarrow \infty} n(a^{\frac{1}{n}} - 1)$ , where  $a > 0$  is a constant. 5%

(c)  $\lim_{x \rightarrow 0} \left( \frac{1}{x} \int_0^{\sin x} f(t) dt \right)$ , where  $f(t)$  is a continuous function. 5%

3. Find a real number  $c$  satisfying the following equality, 10%

$$\int_{-\infty}^c x e^{2x} dx = \lim_{x \rightarrow \infty} \left( \frac{x+c}{x-c} \right)^x.$$

4. Let  $f(x, y) = \begin{cases} \frac{xy}{x^2+y^2}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0). \end{cases}$

(a) Is  $f$  differentiable at  $(0, 0)$ ? 5%

(b) Is  $\frac{\partial f}{\partial x}$  continuous at  $(0, 0)$ ? 5%

5. Verify that the volume of a sphere of radius  $r$  is  $\frac{4}{3}\pi r^3$ . 10%

6. Let  $f(x) = \frac{2(x^2-9)}{x^2-4}$ .

(a) Find all relative extrema of  $f(x)$ . 5%

(b) Determine the concavity of the graph of  $f(x)$  and find its points of inflection. 5%

(c) Find the vertical asymptotes of the graph of  $f(x)$ . 5%

(d) Find the horizontal asymptotes of the graph of  $f(x)$ . 5%

(e) Sketch the graph of  $f(x)$ . 10%

7. Consider the real-valued function  $f(x)$  defined by

$$f(x) = \begin{cases} x \sin \frac{1}{x} & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

(a) Show that  $f(x)$  is continuous at everywhere. 5%

(b) Show that  $f(x)$  is differentiable at everywhere except  $x = 0$ . 10%