※ 考生請注意：本試題 ☑可 □不可 使用計算機

10%(1): Prove that if friction is negligible, the time taken for a particle to slide down any chord of a vertical circle starting from rest at the top of the circle is the same regardless of the chord. (see Fig-1)

10%(2): Considers the system shown in Fig-2, the mass of the three objects and the pulley are m_1, m_2, m_3 and M respectively. Assume that all the contacted surfaces are smooth. If the block of mass m_2 is at rest, (that is $a_2 = 0$). show that the mass of the pulley is

$$M = \frac{3m_1m_3 - m_1m_2 - 4m_2m_3}{m_2 + m_3}$$

10%(3): Consider a small solid ball of radius r rest on the top of a big fixed ball of radius R initially. A small perturbation make the ball begins to move, suppose the motion of the ball is rolls without slipping. Find the position where the small ball just leave the big ball.

10%(4): A uniform disk of radius R is supported by a pivot at point O, a distance r from its center. (see Fig-3)

  (a) If the disk is set into small oscillation, find its period. (5%)

  (b) For what value of r does one obtain the shortest period of oscillation? What is the value of r for the longest period? (5%)

10%(5): For ileal gas, show that for adiabatic process (絕熱過程), the P-V has the relation

$$PV^\gamma = \text{constant}$$

where $\gamma = C_P/C_V$, and C_P is the molar heat capacity at constant pressure, C_V is the molar heat capacity at constant volume.

10%(6): A tank open to the atmosphere contains a liquid of density ρ and has a small hole at the bottom of it. Suppose the cross section areas of the tank and the hole are A_2 and A_1 respectively($A_2 \gg A_1$). At beginning the tank is filled with water at a height H. Show the time required to empty the tank is given by $\frac{A_2}{A_1}\sqrt{\frac{2H}{g}}$
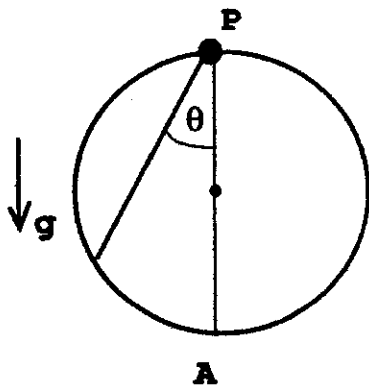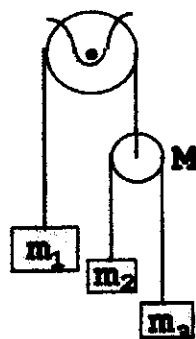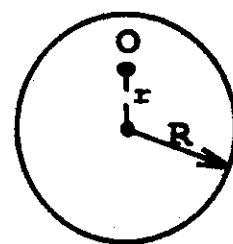

Fig-1

Fig-2

Fig-3

（背面仍有題目,請繼續作答）

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10%(7): Consider a solid insulating sphere of radius R with nonuniform charge density $\rho = Cr$, where r is the distance to the center of the sphere and C is a constant. Find

(a) the total charge contain in the insulating sphere. (4%)

(b) the electric field in the regions (i) $r < R$, (ii) $r > R$. (6%)

10%(8): In the R-L-C series circuit of Fig-4, let $R = 300\ \Omega$, $L = 0.9\ H$, $C = 2.0\ \mu F$. If the circuit connected across an ac source of voltage amplitude 50 V and angular velocity $\omega = 1000\ rad \cdot s^{-1}$. Find

(a)The current amplitude. (2%)

(b)The phase angle between the current and the voltage. (2%)

(c)The voltage amplitude across the resistor. (2%)

(d)The voltage amplitude across the capacitor. (2%)

(e)The voltage amplitude across the inductor. (2%)

10%(9): A particle moves between two fixed solid walls can produces radiations, suppose the particle transits from 1st excited sate(n=2) to ground state, radiate a phonon with frequency ν_1 , and transits from 3rd excited sate(n=4) to ground state, radiate a phonon with frequency ν_2, then what is the value of ν_2/ν_1?

10%(10): In the living creature, the ratio of ^{14}C to ^{12}C is 10^{-13} , and the half life of ^{14}C transmuted into ^{12}C is 5730 years. Now, the ratio of ^{14}C to ^{12}C is 1.25×10^{-14} in an ancient creature, what is the time when the ancient creature died before today?
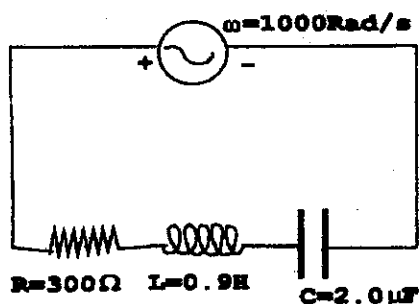

Fig-4