

系所組別： 交通管理科學系乙、丙、丁組

考試科目： 微積分

考試日期： 0306，節次： 2

※ 考生請注意：本試題 可 不可 使用計算機

1. Find $\lim_{x \rightarrow 0} \frac{x^3}{x - \sin x}$. (10%)

2. Find $\lim_{x \rightarrow \infty} \frac{\int_0^x \sqrt{1+t^4} dt}{x^3}$. (10%)

3. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin x \cdot \cos x}{1 - \sin x} dx$. (10%)

4. Evaluate $\int_1^2 \ln(x^2 + 4) dx$. (10%)

5. Evaluate $\int_0^1 \int_x^1 \cos y^2 dy dx$. (10%)

6. Use series to solve the initial value problem

$$y'' + y' = 0, \quad \text{where } y = 0 \text{ and } y' = 1 \text{ at } x = 0.$$

What function does the result represent? (10%)

7. Suppose that \$10,000 is invested for five years at 8%. Find its value of interest is compounded (a) semiannually, (b) monthly, and (c) continuously. (10%)

8. Find the area of the region outside the cardioid $r = 2(1 + \cos \theta)$ and inside the circle $r = 6 \cos \theta$. (10%)9. Suppose that the revenue $r(x) = 9x$ and the cost $c(x) = x^3 - 6x^2 + 15x$, where x represents thousands of units. Is there a production level that maximizes profit $p(x) = r(x) - c(x)$? If so, what is it? (10%)10. Suppose that when a consumer purchases x units of one product, y of another, and z of a third, the utility of the purchases is given by $u = 5x^{\frac{1}{2}}y^{\frac{2}{3}}z^{\frac{1}{4}}$. If the price per unit of the products is \$2, \$5 and \$1, respectively, and the consumer has \$100 to spend, how many units of each product should be purchased to achieve maximum utility? (Hint: consider using $\ln u$.) (10%)