※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

You do not need to calculate the exact number of the answers. However, please simplify your answers as possible as you can. Also, please write the answers in order.

1. **[10 Points]**　Company A determines that the price-demand function for a new book is $p(x)=\dfrac{-x}{500}+20$, with fixed costs of \$12,000 and variable costs of 4.5 dollars per book.
   (a)　[3 points]　Find the profit function F(x).
   (b)　[2 points]　Compute the marginal profit function F'(x).
   (c)　[5 points]　Evaluate F'(3000) and interpret the result.

2. **[15 points]**　Differentiate the following functions:
   (a)　[5 points]　$f(x) = \log(x^3 + 9)$
   (b)　[5 points]　$f(x)=x^5 0.8^x$
   (c)　[5 points]　$f(x) = \tan^5 4x$

3. **[10 points]**　The number of new subscriptions to a newspaper, y, in a month is a function of the amount, x, in dollars spent on advertising in that month, so y=f(x).
   (a)　[3 points]　Interpret the statement f(300)=200
   (b)　[3 points]　Interpret the statement f'(300)=3
   (c)　[2 points]　Use the statements given in part (a) and (b) to estimate f(301) and f(310).
   (d)　[2 points]　Which estimate is more reliable? Why?

4. **[15 points]**　Analyze the graph of $f(x)=\dfrac{2(x^2-9)}{x^2-4}$.
   (a)　[3 points]　Find out the asymptote(s).
   (b)　[12 points]　What are the characteristics (decreasing/ increasing? concave downward/ upward?) of the x-intervals: (-∞, -2), (-2, 0), (0,2), and (2,∞)?

5. **[10 points]**　Find the sum of each infinite series.
   (a)　[5 points]　$\displaystyle\sum_{i=1}^{\infty}\frac{5}{2^n}$　　　　　　(b)　[5 points]　$\displaystyle\sum_{i=0}^{\infty}(\frac{1}{3^n}+\frac{1}{4^n})$

6. **[10 points]**　Define a probability density function of x as

$$f(x)=\begin{cases} kx & 0\le x<2 \\ k(4-x) & 2\le x<4 \\ 0 & \text{otherwise} \end{cases}.$$

   (a)　[5 points]　What is E(x), the expect value of f(x)?
   (b)　[5 points]　What is Var(x), the variance of f(x)?
   　　　　<Note>　　Var(x)= $E(x^2)-[E(x)]^2$

7. **[10 points]**　Suppose company B determines that the price-demand equation for their product, bag, is given by
$$p+2px+x^2 =125 \qquad 0<x\le100,$$
where x represents the demand for the bad in thousand and p represents the price in dollars.

(a) [5 points]　Use implicit differentiation to determine $\dfrac{dp}{dx}$.

(b)　[5 points]　Evaluate and interpret $\dfrac{dp}{dx}$ when x=2.5 and p=19.5.

8. **[10 points]**　A manager at company C finds through data gathered in research that the marginal cost function for a certain type of automobile computer chip made at the facility is given by

$$MC(x) = 6x\sqrt{x^2 + 11}$$

Where x represents the number of auto computer chip produced each hour, and MC(x) represents the marginal cost. The manager also knows that it costs $1932 to manufacture five chips.

(a)　[5 points]　Find the cost function, C(x).

(b)　[5 points]　Determine the fixed costs.

<Note> To avoid any confusion, we will call the arbitrary constant d. Therefore, C(x) + d = $\int MC(x)dx$

9. **[10 points]**　Company D generates income at the rate of 3t thousand dollars per year, where t is the number of years from now.

(a)　[7 points]　Determine the present value of this continuous income stream for the next seven years at 8% compounded continuously.

(b)　[3 points]　Determine the total amount (income plus interest) produced by the company D over this seven-year period.

<Note> If f(t) is the rate of follow function for a continuous income stream, then the present value, P, at annual interest rate, r, compounded continuously for T years is given by $P = \int_0^T f(t)e^{-rt}dt$.

Moreover, $e^{-0.56}$= 0.57 and $e^{0.56}$=1.75.

Reference

1. Bills Armstrong & Don Davis, Brief Calculus—for the Business, Social, and Life Science, 3rd Ed.
2. Hughes-Hallett, Gleason, et al, Applied Calculus, 3rd Ed.
3. Ron Larson & David C. Falvo, Calculus -- An Applied Approach, 8th Ed.