

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Part A Multiple Choice Questions (50 points, 5 points each)

1. Find the values of A and B that make the function $f(x)$ continuous everywhere.

$$f(x) = \begin{cases} (x^2 - 4)/(x - 2), & x < 2 \\ Ax^2 - Bx + 3, & 2 < x < 3 \\ 2x - A + B, & x \geq 3 \end{cases}$$

- (a) $A=1/2, B=1/2$
 (b) $A=5/2, B=1/2$
 (c) $A=-1/2, B=-1/2$
 (d) $A=1/2, B=5/2$
2. For what values of c does the curve have maximum and minimum points $f(x) = 5x^3 + cx^2 + 10x$
- (a) $|c| > 15$
 (b) $|c| > \sqrt{150}$
 (c) $|c| > 1500$
 (d) $|c| > \sqrt{30}$
3. Find the point on the line $y = 4x + 8$ that is closest to the origin.
- (a) $[-32/17, 10/17]$
 (b) $[-34/17, 9/17]$
 (c) $[-32/17, 8/17]$
 (d) $[-31/17, 8/17]$
4. Find the volume of the solid of revolution formed by rotating the region R about the x -axis, R is the region under the curve $y = e^{-0.1x}$ from $x = 0$ to $x = 10$.
- (a) $5\pi(e^{-2} - 1)$
 (b) $10\pi(1 - e^{-2})$
 (c) $5\pi(1 + e^{-2})$
 (d) $5\pi(1 - e^{-2})$
5. Evaluate $\sum_{k=1}^{\infty} \frac{1}{k(k+1)} = \underline{\hspace{2cm}}$
- (a) 0
 (b) 1
 (c) ∞
 (d) -1
6. Calculate $\int_1^4 \int_1^2 \left(\frac{x}{y} + \frac{y}{x}\right) dy dx$
- (a) $\ln \frac{15}{8}$
 (b) e^2
 (c) $\frac{3}{2} \ln 3$
 (d) $\frac{21}{2} \ln 2$

7. Find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{(10x)^{2n}}{(2n)!}$.
- (a) 0
(b) 10
(c) 100
(d) ∞
8. A fast-food restaurant determines the cost model, $C = 0.3x + 4500, 0 \leq x \leq 30000$ and revenue model, $R = \frac{1}{20000}(45000x - x^2)$ for $0 \leq x \leq 30000$ where x is the number of hamburgers sold. Determine the interval on which the profit function is increasing and on which it is decreasing.
- (a) the profit function is increasing on the interval (19500, 30000) and decreasing on the interval (0, 19500)
(b) the profit function is increasing on the interval (0, 12500) and decreasing on the interval (12500, 30000)
(c) the profit function is increasing on the interval (0, 19500) and decreasing on the interval (19500, 30000)
(d) the profit function is increasing on the interval (0, 4500) and decreasing on the interval (4500, 30000)
9. Determine the location of the vertical and horizontal asymptotes of the graph of the function $f(x) = \frac{2x+3}{3x^2+5x-2}$.
- (a) Vertical asymptotes at $x = -2$ and $x = \frac{1}{3}$; horizontal asymptote at $y = 0$
(b) Vertical asymptotes at $x = \frac{2}{3}$; horizontal asymptote at $y = -2$ and $y = \frac{1}{3}$
(c) Vertical asymptotes at $x = -\frac{3}{2}$; horizontal asymptote at $y = 0$
(d) Vertical asymptotes at $x = -2$ and $x = \frac{1}{3}$; horizontal asymptote at $y = \frac{2}{3}$
10. Determine if the series $\sum_{n=1}^{\infty} \frac{(-1)^n}{9n^{1/4}+1}$ converges absolutely, converges, or diverges.
- (a) converges
(b) converges conditionally
(c) diverges
(d) converges absolutely

Part B Please simplify your answers as possible as you can. (50 points)

- [12 points]** The demand function for a product is modeled by $p = 24 - 2\sqrt{x}$, $0 \leq x \leq 144$ where p is the price (in dollars) and x is the number of units. Determine when the demand is elastic, inelastic, and of unit elasticity.
- [10 points]** The rate of change of mortgage debt outstanding for one- to four-family homes in the United States from 2013 through 2015 can be modeled by

$$\frac{dM}{dt} = 600t - 720t^2 + 200e^{-t}$$

Where M is the mortgage debt outstanding (in trillions of dollars) and t is the year, with $t = 0$ corresponding to 2013. In 2013, the mortgage debt outstanding in the United States was \$9000 billion. Please find the average mortgage debt outstanding for 2013 through 2015.

- [20 points, 5 points each]** Evaluate the indefinite integral or definite integral if it converges.
 - $\int \sin(\ln x) dx$
 - $\int x^5 e^{x^3} dx$
 - $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} dx$
 - $\int_1^2 \frac{1}{\sqrt[3]{x}-1} dx$
- [8 points]** A store expects to sell 200 executive desks a year. Each desk costs the store \$400, and there is a fixed charge of \$800 per order. If it costs \$200 to store an executive desk for a year, how large should each order be, and how often should orders be placed to minimize the inventory costs?

Reference

Ron Larson and Tzuwei Cheng (2014), *Calculus: An Applied Approach*

Bill Armstrong and Don Davis (2014), *Brief Calculus for the Business, Social, and Life Sciences*