國立成功大學 106 學年度碩士班招生考試試題

系　　所：企業管理學系

考試科目：微積分

考試日期：0214，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Part A　Multiple Choice Questions (50 points, 5 points each)

1. Find the inflection points for the function $f(x) = 8x + 2 - \sin x, 0 < x < 3\pi$

   (a) $(\pi, 8\pi), (2\pi, 16\pi + 2)$

   (b) $(\pi, 2), (2\pi, 16\pi + 2)$

   (c) $(\pi, 8\pi + 2), (2\pi, 16\pi + 2)$

   (d) $(\pi, 8\pi), (2\pi, 16\pi)$

2. Find the point on line $y = 4x + 8$ that is closest to the origin.

   (a) $\left[\frac{-32}{17}, \frac{8}{17}\right]$

   (b) $\left[\frac{-32}{17}, \frac{10}{17}\right]$

   (c) $\left[\frac{-31}{17}, \frac{8}{17}\right]$

   (d) $\left[\frac{-34}{17}, \frac{9}{17}\right]$

3. Find all the critical numbers of the function $f(x) = \sin^2 x + 2\cos x$.

   (a) $\frac{\pi(2n+1)}{8}, n \in \mathbf{Z}$

   (b) $\frac{\pi(n+1)}{4}, n \in \mathbf{Z}$

   (c) $\frac{n\pi}{2}, n \in \mathbf{Z}$

   (d) none of the these

4. Which of the following functions has a removable discontinuity at c?

   (a) $f(x) = \frac{x^4-1}{x-1}, c = 1$

   (b) $f(x) = [\![\sin x]\!], c = \pi$ where $[\![x]\!]$ is the largest integer function

   (c) $f(x) = \frac{1}{x^2}, c = 0$

   (d) none of the these

5. Calculate $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^4 x \, dx$.

   (a) $\frac{\pi}{512}$　(b) $\frac{3\pi}{512}$　(c) $\frac{\pi}{2}$　(d) none of the these

6. Calculate $\int (9 - x^2)^{1/2} \, dx$

(a) $\frac{1}{2}\left(x\sqrt{9-x^2}+9\sin^{-1}\frac{x}{3}\right)+c$

(b) $\frac{1}{2}\left(x\sqrt{9-x^2}+9\sin^{-1}x\right)+c$

(c) $-\frac{1}{2}\left(x\sqrt{9-x^2}+3\sin^{-1}x\right)+c$

(d) $-\frac{1}{2}\left(x\sqrt{9-x^2}+9\sin^{-1}x\right)+c$

7. Evaluate $\sum_{k=2}^{\infty}\frac{(-2)^{k+1}}{e^k}$.

(a) $-\infty$

(b) 0

(c) $\frac{-8}{e(e+2)}$

(d) $\frac{-6}{e(e+2)}$

8. Find the volume obtained by rotating the curves $x=\theta-\sin\theta,\ y=1-\cos\theta, 0\le\theta\le 2\pi$ about the y-axis.

(a) $6\pi^2$

(b) $5\pi^2$

(c) $4\pi^2$

(d) $3\pi^2$

9. Evaluate $\iint_{\Omega} xy\,dx\,dy$, where the region Ω is the parallelogram bounded by the curves $x^2+y^2=4$, $x^2+y^2=9$, $x^2-y^2=1$, $x^2-y^2=4$.

(a) 13/8

(b) 8/3

(c) 16/7

(d) 15/8

10. Evaluate $\lim_{n\to\infty}\frac{n!}{n^n}$

(a) ∞

(b) 0

(c) $-\infty$

(d) 1

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Part B　Please simplify your answers as possible as you can. (50 points)

1. **[10 points]** Suppose that $f(x)$ is continuous on the closed intervl $[0,1]$ and $0 < f(x) < 1$. Please show that there exists a number $c \in (0,1)$ such that $f(c) = c$.

2. **[10 points]** Please show that $(1+x)^p > 1 + px, \forall x \in (0,\infty)$ where p is any positive real number and $p > 1$.

3. **[10 points]** A business sells 2000 units of product per month at a price of $10 each. It can sell 250 more items per month for each $0.25 reduction in price. What price per unit will maximize the monthly revenue?

4. **[10 points]** A manufacture's production is modeled by the Cobb-Douglas function

$$f(x,y) = 100x^{0.24}y^{0.76},$$

where x represents the units of labor (at $24 per unit) and y represents the units of capital (at $38 per unit). The total costs for labor and capital cannot exceed $100,000. Find the maximum production level for this manufacturer.

5. **[10 points, 5 points each]** Evaluate the definite integral if it converges.

   (a) $\int_0^{\pi/2} \ln(\tan x)\, dx$

   (b) $\int_{-3}^{3} \frac{1}{x(x+1)} dx$

Reference

Ron Larson and Tzuwei Cheng (2014), Calculus: An Applied Approach

Bill Armstrong and Don Davis (2014), Brief Calculus for the Business, Social, and Life Sciences