

國立成功大學
110學年度碩士班招生考試試題

編 號：250

系 所：企業管理學系

科 目：微積分

日 期：0203

節 次：第 3 節

備 註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

一、選擇題:(30 分，每題 6 分)

1. Let f be a real valued function defined on an open interval I . Which of the following statement is TRUE?
- If f is continuous on I , then f is differentiable on I .
 - If f is increasing and differentiable on I then $f' > 0$ on I .
 - If f is twice differentiable on I , and $f''(p) = 0$ for some $p \in I$, then p is an inflection point of f .
 - If $[a, b]$ is a bounded closed subinterval of I , then there exists $c \in [a, b]$ so that $f(x) \leq f(c)$ for all $a \leq x \leq b$.

2. Let $f(x) = x^2 - x - \ln x + 1$. Which of the following statement is FALSE?
- f is increasing on $(1, \infty)$
 - The function has two critical numbers 1 and $\frac{-1}{2}$.
 - $f(x)$ has an absolute minimum at $x = 1$.
 - f is concave upward on $(0, \infty)$.

3. The demand function for a product in NTD is

$$p(x) = 2100 - 0.4x - 0.003x^2.$$

The consumer surplus when the sales level is b is given by the formula

$$CS(b) = \int_0^b (p(x) - p(b))dx.$$

Then $CS(100) =$

- 1000
 - 2000
 - 3000
 - 4000
4. Which of the following infinite series is DIVERGENT?
- $\sum_{n=1}^{\infty} \cos \frac{1}{n}$
 - $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n\sqrt{n}}$
 - $\sum_{n=1}^{\infty} \sin \left(\frac{\pi}{2^n} \right)$
 - $\sum_{n=1}^{\infty} (-1)^{n-1} (\sqrt[n]{n} - 1)$

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5. Let $z = f(x, y)$ be a real valued twice continuously differentiable function defined on a domain in \mathbb{R}^2 and $\Delta = f_{xx}f_{yy} - f_{xy}^2$. Suppose that

$$f_x(a, b) = f_y(a, b) = 0.$$

Which of the following statement is FALSE?

- (a) If $f_{xx}(a, b) = 1$ and $\Delta(a, b) = 3$, then f has a local minimum at $P(a, b)$.
 (b) If $f_{xx}(a, b) = -2$ and $\Delta(a, b) = 16$, then f has a local maximum at $P(a, b)$.
 (c) If $f_{xx}(a, b) = 1$ and $\Delta(a, b) = 0$, then $P(a, b)$ is neither a local maximum of f nor a local minimum of f .
 (d) If $f_{xx}(a, b) = 0$ and $\Delta(a, b) = -5$, then $(a, b, f(a, b))$ is a saddle point of the graph $z = f(x, y)$.

二、填充題 (35 分，每個空格 7 分)

1. $\lim_{x \rightarrow 0^+} (1 + \sin 3x)^{\frac{1}{x}} = \text{A}$.

2. The slope of the tangent line to the curve

$$\tan y = \frac{1-x}{1+x}$$

at $P\left(0, \frac{\pi}{4}\right)$ is B.

3. $\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}} dx = \text{C}$. Write DNE if the limit does not exist.

4. If R is the radius of convergence of the power series

$$\sum_{n=1}^{\infty} \left(1 + \frac{1}{n}\right)^{-n^2} x^n,$$

then $R = \text{D}$.

5. Suppose that X and Y are continuous random variable with joint density function

$$f(x, y) = \begin{cases} \frac{c}{(x^2 + y^2)^{\frac{3}{2}}} & \text{if } x^2 + y^2 \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

Then $c = \text{E}$.

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三、計算題:(35 分)，請詳列各題之計算過程，否則將不予計分。

1. (10 分) An electronics store has been selling 10 iPhone XII at 2,0000 NTD each. A market survey shows that for each 1000 NTD rebate offered to buyers, the number of iPhone XII sold will increase 2 by a week. How large a rebate should the store offer to maximize its revenue?

2. (10 分) Assume that there are two commodities with amounts x, y and the consumer has utility function

$$U(x, y) = xy.$$

Maximize the utility function subject to the consumer's budget constraint $3x + 4y = 5$.

3. (15 分) The government of a certain country decides to introduce new currency by having the banks replace the old bills with new ones as long as the old currency comes into the bank. Let $x(t)$ be the amount of the new currency in circulation with unit in billion NTD at time t . A mathematical model that represents the flow of the new currency into circulation in that country is given by

$$\frac{dx}{dt} = 0.002(40 - x), \quad x(0) = 0.$$

Find $x(t)$.