國立成功大學

111學年度碩士班招生考試試題

編　號：245

系　所：企業管理學系

科　目：微積分

日　期：0220

節　次：第 3 節

備　註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Part A: Multiple-choice questions, 32 points, 8 points each. One answer or four answers is possible. Select all the correct items to get the points.

1. For a quadratic function $y = f(x) = x^2 + bx + c$, which of the following statements are TURE?

   A. When $x = -\frac{b}{2}$, y reaches its minimum $c - \frac{b^2}{4}$.

   B. If $\alpha < \beta$, $\frac{f(\alpha)+f(\beta)}{2} < f\left(\frac{\alpha+\beta}{2}\right)$

   C. The tangent line of $f(x)$ at $(a, f(a))$ is $y = (2a + b)(x - a) + c$

   D. If $h < c$ and $m = b + 2\sqrt{c - h}$, then $y = mx + h$ is a tangent line of $f(x)$.

2. Which of the following formulas have the same value as the definite integral $\int_0^1 x^2 \, dx$?

   A. $\lim\limits_{n \to \infty} \frac{1}{3} \sum_{k=1}^{n} \frac{(k+1)^3 - k^3}{n^3}$

   B. $\lim\limits_{n \to \infty} \frac{1}{3} \sum_{k=0}^{n-1} \frac{(k+1)^3 - k^3}{n^3}$

   C. $\lim\limits_{n \to \infty} \frac{1}{n} \sum_{k=1}^{n} \frac{k^2}{n^2}$

   D. $\lim\limits_{n \to \infty} \frac{1}{n} \sum_{k=0}^{n-1} \frac{k^2}{n^2}$

3. If $f(x, y) = y e^{-x}$, which of the following statements are TURE?
   A. $f_x(x, y) + f_y(x, y) = 0$
   B. $f_x(x, y) + y \cdot f_y(x, y) = 0$
   C. $f_{xx}(x, y) - f(x, y) = 0$
   D. $f_{yy}(x, y) = 0$

4. For the function $f(x) = \ln(1 + x^2)$, which of the following statements are TRUE?

   A. $f'(x) = \frac{1}{1+x^2}$

   B. $f'(x) = 2 \sum_{k=0}^{\infty} (-1)^k x^{2k+1}$ for $|x| \le 1$

   C. $f(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+1} x^{2k+2}$ for $|x| < 1$

   D. $f(1) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k+1}$

Part B: Fill in the blanks, 28 points, 4 points each. Derivations are not required.

5. Find the approximation of $\sqrt[3]{6}$ by using Newton's method with the function $f(x) = x^3 - 6$. The iterative formula of Newton's Method is $x_{n+1} =$ ___(5a)___. If $x_0 = 1$, $x_2 =$ ___(5b)___.

6. A company produces the VR-glasses, and the cost is $1000 + 80x + 2x\sqrt{x}$ when producing x units. The average cost per unit reaches its minimum ___(6a)___ when they produce ___(6b)___ units.

7. By substituting $u = \ln x$ in the indefinite integral $\int \frac{(\ln x)^2}{x^3} dx$, the indefinite integral will be

   reformulated as $\int g(u)\, du$, where $g(u) =$ ___(7a)___ and $\int \frac{(\ln x)^2}{x^2} dx =$ ___(7b)___.

8. Evaluate the double integral $\int_0^1 \int_0^{x^2} x\sqrt{y}\, dy\, dx$ ___(8)___.

Part C: Answer the following questions, 40 points, 10 points each. Derivations are required. Simplify your answer as possible as you can.

9. Compute the indefinite integral $\int e^{2x} \cdot \sin(3x) \cdot dx$.

10. Given a function $f(x)$ where $f'(x) = \frac{1}{1+x^3}$ and use $f(x)$ to express the indefinite integral

   $\int \frac{1}{8+(3x-1)^3} dx$.

11. A new study shows that the rate of percentage at which the target population hears about a new product is assumed to be proportional to the days (t) after the releasing date ($t = 0$) times the number of people who do not yet hear about the new product. A company releases a new product which the target population knows nothing about it. One day after the releasing date ($t = 1$), 10% of the population has heard about the product. Write down the differential equation model for the percentage (y), and use the above information to estimate the percentage in the target population has heard about the product two days after the releasing date.

12. For the function $f(x, y) = x^2 + y^3 - 2xy - y + 3$, determine whether there is a relative maximum, a relative minimum, a saddle point, or insufficient information to determine the nature of the function $f(x, y)$ at each critical point.