國立成功大學

112學年度碩士班招生考試試題

編　號：243

系　所：企業管理學系

科　目：微積分

日　期：0207

節　次：第 3 節

備　註：不可使用計算機

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Part A: Multiple-Choice Questions (30 points, 10 points each)

1. Find the correct statement(s).

   (a) Given functions f, g such that $f(x)$ and $f(x)+g(x)$ are both continuous at $x = 0$, then $g(x)$ is also continuous at $x = 0$.

   (b) Given functions f, g such that $f(x)$ and $f(x) \cdot g(x)$ are both continuous at $x = 0$, then $g(x)$ is also continuous at $x = 0$.

   (c) Given function f such that $|f(x)|$ is continuous at $x = 0$, then $f(x)$ is also continuous at $x = 0$.

   (d) Equation $x^3 + x^2 + x = 1$ has exactly one solution.

   (e) Equation $x^2 = 2^x$ has exactly two solutions.

2. Given function $f(x) = xe^{-x/2}$.

   (a) $f(1) < f(3)$.

   (b) Function $f(x)$ is increasing on interval $(0, 1)$.

   (c) Function $f(x)$ reaches absolute maximum when $x = 2$.

   (d) Graph of $f(x)$ has an inflection point when $x = 3$.

   (e) Function $f(x)$ is concave upward on interval $(4, \infty)$.

3. Given function $f(x, y) = 8x^3 - 12xy + y^3$.

   (a) Function $f(x, y)$ has maximum rate of change at point $(-\frac{1}{2}, 0)$ in direction $\langle \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$.

   (b) Function $f(x, y)$ has maximum rate of change at point $(-\frac{1}{2}, 2)$ in direction $\langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \rangle$.

   (c) Point $(0, 0)$ is a saddle point of $f(x, y)$.

   (d) Function $f(x, y)$ reaches local minimum at $(1, 2)$.

   (e) Function $f(x, y)$ reaches local maximum at $(-1, -2)$.

Part B: Fill in the Blanks (40 points, 8 points each)

4. Evaluate the limit. $\lim_{x \to 0} \dfrac{1 - \cos 2x}{\sin 3x \cdot \tan 4x} =$ _____

5. Let $f(x) = \displaystyle\int_{\frac{1}{x}}^{x^2} \sqrt{1 + t^3}\, dt$. Evaluate $f'(1) =$ _____.

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6. Evaluate the integral. $\int_1^e (\ln x)^2\,dx =$ _____

7. Evaluate the limit. $\lim_{x\to\infty}(3^x + x^3)^{3/x} =$ _____

8. Given the Maclaurin series as below. Find the value of $a_{101} =$ _____.

$$\sin(x)\cos(x) = a_0 + a_1x + a_2x^2 + \cdots + a_{101}x^{101} + \cdots$$

Part C: Show All Your Work (30 points, 10 points each)

9. The diameter ($d = 2r$) and height (h) of a cylinder was both measured to be 10 cm with a possible error ($\Delta d, \Delta h$) at most 0.05 cm. Use the differential to approximate the maximum error (ΔA) in computing the surface area ($A = 2\pi r^2 + 2\pi rh$).

10. Evaluate the integral by changing the order. $\int_0^1 \int_{2y}^2 e^{-x^2}\,dx\,dy$

11. If a company is planning to promote a product by spending x thousand dollars on television commercials and y thousand dollars on internet advertising, then the revenue is expected to increase by

$$f(x,y) = \frac{160\,x}{x+3} + \frac{150\,y}{y+5}$$

thousand dollars. Use the method of Lagrange multiplier to maximize the revenue under the budget constraint

$$g(x,y) = x + y = 100.$$