國立成功大學

113學年度碩士班招生考試試題

編　號：236

系　所：企業管理學系

科　目：微積分

日　期：0202

節　次：第 3 節

備　註：不可使用計算機

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Throughout the exam, you may leave all transcendental constants and their combinations, such as e^7, $\ln(\sqrt{7})$, ... etc, as they are. (i.e. without their numerical values.)

Part A: Multiple Choice. Pick *ONE* correct answer in each problem.

1. (5 points) Select the correct statement from the followings:

   (a) If a differentiable function f has zero derivative everywhere on its domain, f must be constant.

   (b) A real cubic (degree 3) polynomial may have exactly 2 real roots.

   (c) For differentiable functions f,g with $f' > g'$, we must have $f > g$.

   (d) A real polynomial of degree 7 has at least one real root.

   (e) All of the above.

2. (5 points) Let $f(x, y)$ be a function on $\mathbb{R}^2$ whose first and second derivatives exist and are continuous everywhere. Given a point P on $\mathbb{R}^2$ such that

$$f_x(P) = f_y(P) = 0, \quad f_{xy}(P) = 3$$

   Here, the subscripts indicate the derivative(s) of f with respect to that(those) variable(s), in the order displayed. Which condition below makes P a point where f has local *maximum* there?

   (a) $f_{xx}(P) = -1, f_{yy}(P) = -2$.

   (b) $f_{xx}(P) = 2, f_{yy}(P) = 8$.

   (c) $f_{xx}(P) = 0, f_{yy}(P) = -3$.

   (d) $f_{xx}(P) = -6, f_{yy}(P) = -8$.

   (e) $f_{xx}(P) = -2, f_{yy}(P) = \frac{1}{8}$.

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3. (5 points) The value of

$$\sum_{n=0}^{\infty} \frac{2^n}{n!(n+2)}$$

is

(a) $-\frac{2}{3}$.

(b) $\frac{e^2+1}{4}$.

(c) $\frac{1}{2}e^4$.

(d) -1

(e) $\frac{e^2-1}{4}$.

Hint: Consider $f(x) = xe^x$

4. (5 points) How many of the following integrals is(are) finite?

$$\int_1^{\infty} e^{\frac{-1}{x^2}}\,dx,\quad \int_0^9 \frac{1}{x-9}\,dx,\quad \int_3^{\infty} \frac{1}{x^2-\ln x}\,dx,\quad \int_3^8 \frac{1}{\sqrt{x^2-9}}\,dx,\quad \int_2^3 e^{\sqrt{x^3-2}}\,dx$$

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

Part B: Fill in the blanks. No work is needed to be shown. Only the final answer is graded *without* partial credit.

5. (10 points)

$$\int_0^1 e^x \cos x\,dx = \_\_\_\_\_.$$

6. (10 points) Given

$$F(x) = \frac{1}{x}\int_0^{2x} \frac{t}{e^{3t}-1}\,dt,$$

compute

$$\lim_{x\to 0} F(x) = \_\_\_\_\_.$$

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7. (10 points) The equation of tangent line at $(0,1)$ to the curve given by

$$\sin(xy) - 2\cos(x^2) = 2e^y - 2e - 2y$$

is

$$y = \_\_\_\_\_ x + \_\_\_\_\_.$$

8. (10 points)

$$\int_0^3 \int_{\frac{1}{3}y}^1 \ln(3x^2+1)\, dxdy = \_\_\_\_\_.$$

Part C: Free response problems. Make sure to show ALL your work to receive full credit(s).

9. (20 points) Sketch the graph of the function

$$f(x) = \frac{x^2+4x+7}{x+3}$$

on the entire $\mathbb{R}$, wherever it is defined. Make sure to indicate all the critical point(s), point(s) of inflection, horizontal and vertical asymptotes, and correct monotonicities/concavities.

10. (10 points) Given a product with revenue function

$$R(x) = xe^{-x^2+x},$$

for $x > 0$, indicate the range of x where the demand is *elastic*.

11. (10 points) A baseball player signs a 700 million dollar contract that pays him 2 million dollars per year from 2024 to 2034, and then 68 million dollars per year from 2034 to 2044. Suppose that the salary is paid continuously and the economy has an annual inflation rate of 3% that also occurs continuously, what is the *present value* of this contract at the beginning of 2024?