

國立成功大學

115學年度碩士班招生考試試題

編 號：166

系 所：企業管理學系

科 目：微積分

日 期：0204

節 次：第 3 節

注 意：1.不可使用計算機
2.請於答案卷(卡)作答，於
試題上作答，不予計分。

Part A: Multi-Select/Multiple-Choice Questions. Select all that apply.

1. (8 points) Which of the following statements are true?
- (A) If $\lim_{x \rightarrow 3} f(x) = 4$ and $\lim_{x \rightarrow 3} g(x) = 0$, then $\lim_{x \rightarrow 3} [f(x)/g(x)]$ does not exist.
 - (B) If $\lim_{x \rightarrow 3} f(x) = 0$ and $\lim_{x \rightarrow 3} g(x) = 0$, then $\lim_{x \rightarrow 3} [f(x)/g(x)]$ does not exist.
 - (C) If neither $\lim_{x \rightarrow a} f(x)$ nor $\lim_{x \rightarrow a} g(x)$ exists, then $\lim_{x \rightarrow a} [f(x) + g(x)]$ does not exist.
 - (D) If $\lim_{x \rightarrow a} f(x)$ exists but $\lim_{x \rightarrow a} g(x)$ does not exist, then $\lim_{x \rightarrow a} [f(x) + g(x)]$ does not exist.
2. (8 points) Which of the following statements are true?
- (A) If $f''(3) = 0$, then $(3, f(3))$ is an inflection point of the curve $y = f(x)$.
 - (B) There exists a function f such that $f(3) = -3$, $f(6) = 0$, and $f'(x) > 1$ for all x .
 - (C) There exists a function f such that $f(x) > 0$, $f'(x) < 0$, and $f''(x) > 0$ for all x .
 - (D) There exists a function f such that $f(x) < 0$, $f'(x) < 0$, and $f''(x) > 0$ for all x .
3. (4 points) If $g(x)$ is continuous for all real values of x , then $\int_{b/3}^{a/3} g(3x) dx =$
- (A) $\int_b^a g(x) dx$
 - (B) $3 \int_b^a g(x) dx$
 - (C) $\frac{1}{3} \int_b^a g(x) dx$
 - (D) $\frac{1}{3} \int_{3b}^{3a} g(x) dx$
4. (5 points) What is the power series centered at 0 and the interval of convergence for the function $1/(x+2)$.
- (A) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} x^n, [-2, 2]$
 - (B) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n} x^n, (-2, 2)$
 - (C) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} x^n, [-2, 2]$
 - (D) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n+1}} x^n, (-2, 2)$

Part B: No partial credit. Showing your work is not necessary, only the answer will be graded.

5. (6 points) If the equation of the tangent line to the curve $x^2 + 2xy + 4y^2 = 12$ at the point $(2, 1)$ is $y = ax + b$, find (a, b) .
6. (6 points) Two cars leave an intersection at the same time. The first car is going due east at the rate of 40 mph (miles per hour) and the second is going due south at the rate of 30 mph. How fast is the distance between the two cars increasing when the first car is 120 miles from the intersection?
7. (5 points) Let f be a function that has derivatives of all orders for all real numbers. Assume $f(0) = 1$, $f'(0) = 6$, $f''(0) = -4$, and $f'''(0) = 30$. Find the third-degree Taylor polynomial for f about $x = 0$.
8. (24 points, 6 points each) Evaluate the following integrals. If the integral does not exist, write DNS.

(A) $\int \ln(2x) dx$

(B) $\int x^3 e^{x^2} dx$

(C) $\int_0^2 \frac{1}{x-1} dx$

(D) $\int_e^\infty \frac{1}{x(\ln x)^2} dx$

Part C: Free-response questions. Show all your work for full credit.

9. (4 points) Use the linear approximation of the function $f(x) = x^3$ at $a = 5$ to approximate the number $(5.2)^3$.
10. (10 points) Evaluate $\int_0^2 \int_0^{4-x^2} \frac{x e^{2y}}{4-y} dy dx$.
11. (10 points) Find the relative extrema of $f(x, y) = x^3 - 3x - y^2 + 4y$.
12. (10 points) Find the solution of the initial-value problem

$$\frac{dy}{dx} = x^3 - x^3 y, \quad y(0) = 3.$$