(丙)

國立成功大學 79 學年度企管研究所考試(微積分 試題) 共 2 頁 第 1 頁

1. Find the following limits if exist. (24%)

(a) $\lim_{x\to 0} \frac{x^2}{\sin x}$ (b) $\lim_{n\to\infty} n^{1/n}$

(c) $\lim_{x\to 0^+} \sin x \ln \sin x$ (d) $\lim_{(x,y)\to(0,0)} \frac{x^2 y}{x^4+y^2}$

(e) $\lim_{x\to\infty} \left(\frac{x^2}{x-1} - \frac{x^2}{x+1}\right)$

(f) $\lim_{x\to 2^-} [x] - x^2$, where $[\ ]$ is the greatest integer function.

2. Find $\int_{-\infty}^{\infty} \frac{1}{e^x + e^{-x}}\, dx$ (6%)

3. Find the volume of the resulting solid by revolving the region (in the first quadrant bounded by the graphs of $y = \frac{1}{8}x^3$ and $y = 2x$) about the y-axis. (10%)

4. Use a Taylor polynomial to approximate $\cos 61°$ and estimate its accuracy. (10%)

5. Find $\int_0^2 r(t)\, dt$ if $r(t) = 12t^3\,\mathbf{i} + 4e^{2t}\,\mathbf{j} + (t+1)^{-1}\,\mathbf{k}$

6. Find $\int_0^4 \int_{\sqrt{y}}^2 y \frac{\cos x^5}{} dy\, dx$ (10%)

7. Find $\iint_R e^{(y-x)/(y+x)}\, dx\, dy$ for the region R in the xy-plane bounded by the trapezoid with vertices (0,1), (0,2), (2,0), and (1,0). (10%)

278

8. Find the interval of convergence of the series

$$\sum_{n=0}^{\infty} (-1)^n \frac{1}{n+1} (x-3)^n \qquad (10\%)$$

9. Use the gradient to find the directional derivative of f at $P(1,2)$ in the direction from $P(1,2)$ to $Q(2,5)$, (10%)

$$f(x,y) = x^2 - 4xy$$