

1.  $\lim_{n \rightarrow \infty} \frac{e^{\frac{1}{n}} + e^{\frac{2}{n}} + \dots + e^{\frac{n}{n}}}{n} = ?$  (10)

2. Evaluate  $\int_0^2 \frac{dx}{(x-1)^2}$ . (10)

3. Find the average distance from the origin to the parabolic arc  $y = x^2$  for  $x \in [0, 1]$ . (10)

4. Let  $f(x, y, z) = xy^2 + y^2z^3 + z^3x$ . In what direction from the point  $P(2, -1, 1)$  does the function  $f$  increase the most rapid? and what direction  $f$  decrease the most rapid? Give your answer in the form of unit vector. (10)

5. Let  $f(x) = x^2$  for  $x \leq 1$ , and  $f(x) = x$  for  $x > 1$ . Does the mean value theorem hold for  $f$  in the interval  $[\frac{2}{3}, 2]$ ? why or why not. (10)

6. Test the following series for convergence or divergence.

(a)  $\sum_{k=2}^{\infty} \frac{\ln \sqrt{k}}{k}$ . (5)

(b)  $\sum_{k=1}^{\infty} \frac{1}{(2k-1)2k}$ . (5)

7. For what value of  $r$  does  $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^r}$  diverge? (10)

8. Approximate  $\int_0^1 e^{-x^2} dx$  to two decimal place. (10)

9. Population tends to grow with time at a rate roughly proportional to the population present. In 1980, the population was approximately 10 million and in 1990, was approximately 20 million. Use this information to estimate the year in which the population may be expected to be approximately 40 million. (10)

10. A real estate office handles 80 apartment units. When the rent of each unit is \$6000 per month, all units are occupied. However, for each \$200 increase in rent, one of the units becomes vacant. Each occupied unit requires an average of \$600 per month for service and repairs. What rent of each unit should be charged to have the most profit? (10)