

一. Calculate the following problems:  
(25%)

(1)  $\lim_{x \rightarrow 0^+} (x + \cos 2x)^{\csc 3x}$

(2)  $\int_{-2}^2 (2 + x \sqrt{1+x^2+x^4}) dx$

(3)  $\int_{1/2}^{11/2} 2 \{2x + 1\} dx$ ,  $\{x\}$  is the greatest integer function.

(4)  $\lim_{n \rightarrow \infty} \left( \frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \dots + \frac{n}{n^2+4n^2} \right)$

(5) Let  $f$  be a real-valued function defined by

$$f(x) = \begin{cases} x \ln |x|, & \text{if } x \neq 0 \\ a, & \text{if } x = 0, \end{cases}$$

What value of  $a$  will make  $f$  continuous?

二. For given  $a_i \geq 0, i = 1, 2, \dots, n$ , find the maximum and minimum values  
(10%)  
of the function  $f(x_1, \dots, x_n) = \sum_{i=1}^n a_i x_i$  subject to the constraints  
 $\sum_{i=1}^n x_i = 1$ .

三. If  $f(x) = x e^{x^2}$ , find the 99th derivative of  $f(x)$  at  $x=0$ .  
(10%)

四. Suppose  $f(x) = xg(x)$ , where  $g(x)$  is continuous at  $x = 0$ , show that  
(10%)  
 $f$  is differentiable at 0 and find  $f'(0)$ .

五. Find the area of the surface generated by revolving the curve  
(10%)  
 $x = \sqrt{2y - 1}, 1/2 \leq y \leq 1$ , about the  $y$ -axis.

六. Is the improper integral  $\int_0^{\infty} \frac{x^2}{1+x^2+x^4} dx$  convergent or divergent?  
(10%)  
Prove your answer.

七. (1) Calculate  $\int_0^{\infty} e^{-x^2} dx$ .  
(15%)

(2) From (1), find  $\int_0^{\infty} x^2 e^{-x^2} dx$ .

(3) Verify  $\int_{-\infty}^{\infty} e^{-x^2/2} dx = \sqrt{2\pi}$ .

八. Find  $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \exp \left\{ - (1/2)(x^2+y^2) \left[ 1 - \frac{xy}{(1+x^2)(1+y^2)} \right] \right\} dx dy$ .  
(10%)