(丙,丁組)

1. If $x > 0$ and $x \neq 1$, prove that $1-\frac{1}{x} < \ln x < x-1$. (10%)

2. Define $\Gamma(x)=\int_o^\infty u^{x-1}e^{-u}du$ for x>0.
Show that (a) $\Gamma(x)=2\int_o^\infty u^{2x-1}e^{-u^2}du$.
(b) $\Gamma(x)=\int_o^1\left[\ln(\frac{1}{u})\right]^{x-1}du$. (15%)

3. Find the following limits :
(1) $\underset{n\to\infty}{Lim}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\cdots+\frac{n}{n^2+n^2}\right]$
(2) $\underset{k\to\infty}{Lim}\sum_{n=1}^{k}\frac{1}{n(n+1)(n+2)}$ (10%)

4. Let $f_n(x)=(\sin nx)/n$, and for each fixed real x, let $f(x)=\lim_{n\to\infty}f_n(x)$. Show that $\lim_{n\to\infty}f_n'(0)\neq f'(0)$. (10%)

5. (a) If 0<x<1, prove that $(1+x^n)^{1/n}$ approaches to a limit as $n\to\infty$ and compute this limit.
(b) Given a>0, b>0, compute $\lim_{n\to\infty}(a^n+b^n)^{1/n}$. (15%)

6. Let $g(x)=xe^{x^2}$ and let $f(x)=\int_1^x g(t)(t+\frac{1}{t})dt$. Compute the limit of $f''(x)/g''(x)$ as $x\to+\infty$. (10%)

7. (a) Show that $\int_o^\pi xf(\sin x)dx=\frac{\pi}{2}\int_o^\pi f(\sin x)dx$.
(b) Use (a) to deduce $\int_o^\pi\frac{x\sin x}{1+\cos^2 x}dx=\pi\int_o^1\frac{1}{1+x^2}dx$. (15%)

8. A truck is to be driven 300 miles on a freeway at constant speed of miles per hour. Speed laws require $30\leq x\leq 60$. Assume that fuel costs 30 cents per gallon and is consumed at the rate of $2+x^2/600$ gallons per hour. If the driver's wages are D dollars per hour and if he obeys all speed laws, find the most economical speed and the cost of trip if (a) D=1 (b) D=2. (15%)