

一、Let the function $f(x)$ be defined as:

$$f(x) = 3 + |ax - 1| + |x + 1|$$

(1) For what value of a would $f(x)$ have a unique inverse?

(2) Find the derivative function of the inverse of $f(x)$.

(8%)

二、In a particular manufacturing process, it is known that the number of rejects y depends on the total daily output x , that is, $y = y(x)$. The firm makes a profit A for each item sold, but loses $A/3$ for each reject. Suppose that

$$y = \begin{cases} \frac{x}{101-x} & \text{for } x \leq 100 \\ x & \text{for } x > 100 \end{cases}$$

what should be the daily output be in order to maximize profits?

(10%)

三、Graph the function $f(x) = x^5/5 - x^3/3$.

(13%)

四、(1) Find $\lim_{x \rightarrow \infty} x \ln \left[\frac{x+1}{x-1} \right]$.

(2) If $a_1 = \sqrt{2}$ and $a_{n+1} = \sqrt{2 + \sqrt{a_n}}$ for $n \geq 1$. Find $\lim_{x \rightarrow \infty} a_n$.

(10%)

五、Find:

(1) $\lim_{x \rightarrow \infty} \left[\frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \dots + \frac{1}{2n} \right]$.

(2) $\lim_{x \rightarrow \infty} \left[\frac{n+1}{n^2} + \frac{n+2}{n^2} + \dots + \frac{2n}{n^2} \right]$.

(10%)

六、Find the area bounded by the curves $y = e^{x/2}$ and $y = x^2$.

(6%)

七、Calculate the following problems:

(1) $\int_0^1 x(1+x)^{1/2} dx$ (2) $\int_0^1 \sin^{-1} x dx$ (3) $\int (1+x^2)^{1/2} dx$

(15%)

八、Apply the method of Lagrange to show that

$$(x_1^2 x_2^2 x_3^2)^{1/3} \leq \frac{1}{3}(x_1^2 + x_2^2 + x_3^2)$$

(10%)

九、 Calculate the following improper integration:

$$(1) \int_{-\infty}^{\infty} \frac{1}{\pi(1+x^2)} dx$$

$$(1) \int_{-\infty}^{\infty} \frac{xe^x}{(1+e^x)^2} dx$$

(10%)

十、 Prove: $\frac{1}{n} \sum_{i=1}^n \log x_i \leq \log \left(\frac{1}{n} \sum_{i=1}^n x_i \right)$ where $x_i > 0$ for each i .

(8%)