

註：每題 10 分，共計 10 題。可不按題號作答，但每一題均應寫出題號。

(1) Suppose that $f(x) = x^2$. Find two different functions g of the form $g(x) = Ax + B$ such that $g(f(x)) = f(g(x))$ for all x .

(2) Two cubes have total volume 250 cm^3 . What is the maximum possible surface area they can have? The minimum? Verify the nature of the extrema that you find.

(3) A triangle has two sides of length 1 and the angle between these two sides is θ . Express the area of the triangle as a function of θ , then find the maximum possible area of such a triangle using calculus.

(4) Use the mean value theorem to prove that $\sin x < x$ if $x > 0$.

(5)(a) Evaluate $\lim_{x \rightarrow \infty} e^{-x^2} \int_0^x e^{t^2} dt$. (b) $\int \sin 2x \sin x dx$

(6) The region R in the first quadrant is bounded above by the graph of $y = \sqrt{\sin x}$ and below by the x -axis, $0 \leq x \leq \pi$. When it is rotated around the x -axis it generates a solid of volume V . Find V .

(7) A certain differentiable function $y = y(x)$ satisfies the differential equation $x^2 \frac{dy}{dx} = y^2$, and it is also known that $y(x) \rightarrow 2$ as $x \rightarrow +\infty$. Find the formula for $y(x)$.

(8) Sketch the graph of the polar equation $r = -2 + 4 \cos \theta$. Then find the area enclosed by the small loop.

(9) Evaluate

$$\iint_D \frac{1}{x^2 + y^2 + 1} dA$$

where D is the circular disk in the plane centered at the origin and with radius $a > 0$.

(10) My volume is $\int_0^{2\pi} \int_0^{1 + \cos \theta} (r + r^3) dr d\theta$. What am I?