注意:未寫明演算過程者不予計分

1. Suppose that $\lim_{x\to x_0} f'(x)$ exists. Does it follow that $f(x)$ is differentiable at x_0 ? Give a proof to show that the statement is correct or produce a counterexample to show that it is false. (6分)

2. Determine if the following limits exist:

(a). $\lim_{(x,y)\to(0,0)} f(x,y) = \dfrac{xy}{x^2+y^2}$ (5分)

(b). $\lim_{(x,y)\to(0,0)} f(x,y) = \dfrac{x^2y}{x^4+y^2}$ (5分)

3. Suppose that $f(x)$ is differentiable on $(0,\infty)$ and $f'(x)\to 0$ as $x\to\infty$. Let $g(x)=f(x+1)-f(x)$. Prove that $g(x)\to 0$ as $x\to\infty$. (7分)

4. Consider the sequence $\{f_n(x)\}_{n=1}^{\infty}$, where $f_n(x)=nx/(1+nx^2)$, $x\geq 0$. Find the limit of $\int_1^2 f_n(x)dx$ as $n\to\infty$. (7分)

5. For what values of a and b is the function

$$f(x)=\frac{1}{x^2+ax+b}$$

bounded on the interval $[-1,1]$? Find the absolute maximum on that interval. (15分)

6. Find (a) $\lim_{x\to 0^+}(\sin x)^x$ (5分)

(b) $\lim_{x\to 0^+}(e^{-1/x}/x)$ (5分) (c) $\lim_{x\to 0}(1+ax)^{1/x}$ (5分)

7. Find (a) $\int\int_D e^{y^2}dxdy$, where D is the region in the first quadrant bounded by $x=y$ and $y=1$. (7分)

(b) $\int\int_D xydxdy$ and D be the region $D=\{(x,y)|x^2+y^2\leq 1, x\geq 0, y\geq 0\}$. (7分)

8. Find the minimum distance from the origin to the curve of intersection of the surfaces $z(x+y)=-2$ and $xy=1$. (10分)

9. (a) Suppose that $f(x)$ is monotone and its derivative $f'(x)$ is Riemann integrable on $[a,b]$. Let $g(x)$ be continuous on $[a,b]$. Show that there exists a number c, $a\leq b\leq c$, such that

$$\int_a^b f(x)g(x)dx = f(a)\int_a^c g(x)dx + f(b)\int_c^b g(x)dx \qquad (10分)$$

(b) Deduce from (a) that for any $b>a>0$,

$$\left|\int_a^b \frac{\sin x}{x}dx\right| \leq \frac{4}{a} \qquad (6分)$$