

注意：(1)請按題號作答；(2)答案務必要寫明演算過程且化簡，否則不予計分。

1. (10分) A motorcyclist riding along the road  $y = x^2$  is at point  $(a, b)$  when she sees a deer standing at point  $(1, -4)$  illuminated by her headlight. Find  $(a, b)$ .

2. (10分) Find the number whose existence is guaranteed by the mean value theorem when that theorem is applied to  $f(x) = \sin x$  on the interval  $(0, \pi)$ .

3. (10分) Given:  $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{1}{n} \cdot \sin\left(\frac{\pi i}{n}\right) = \int_0^1 f(x) dx$ . Find the

formula of the function  $f$ , then use the fundamental theorem of calculus to evaluate the integral.

4. (10分) Find the volume generated when the region bounded by the  $y$ -axis, the graph of  $xy^3 = 1$ , and the horizontal lines  $y = 1$  and  $y = 2$  is rotated around the  $y$ -axis.

5. (10分) Evaluate  $\lim_{n \rightarrow \infty} \frac{e^{2/n} + e^{4/n} + e^{6/n} + \dots + e^{2n/n}}{n}$ .

[Suggestion: Think about Riemann sums.]

6. (20分) Evaluate

(a)  $\int \frac{1}{1 + e^x} dx$     (b)  $\int \sqrt{1 + \cos x} dx$     (c)  $\int \sqrt{1 + e^x} dx$     (d)  $\int \sqrt{1 + \sqrt{x}} dx$

7. (10分) Let  $a_1 = 1$  and, for  $n \geq 1$ , let  $a_{n+1} = 1 - \frac{1}{2}a_n$ . Find the limit of the sequence  $\{a_n\}$  (or show that the limit does not exist).

8. (10分) A solid has its base in the  $xy$ -plane, bounded by the graphs of  $y = x^2$  and  $x = y^2$  for  $0 \leq x \leq 1$  and  $0 \leq y \leq 1$ . Its height at the point  $(x, y)$  is  $xy$ . Find its volume.

9. (10分) Evaluate

$$\iint_D (x^2 + y^2)^{1/4} dA \quad \text{where } D \text{ is the disk } x^2 + y^2 \leq 1.$$