壹、選擇題: 下列各題均只有一個最佳答案，每題 5 分，共 60 分。

1. Consider a function $z = f(x_1, x_2, x_3)$. If z has an absolute maximum, then

   I. f is strictly convex.
   II. f is strictly concave.
   III. d^2z is everywhere positive definite.
   IV. d^2z is everywhere negative definite.

   The correct answer is (A) I and III, (B) I and IV, (C) II and III, (D) II and IV, (E) none of them.

2. $\lim_{x\to 0}(Cosx)^{1/x^2} = ?$ (A) 1, (B) ∞, (C) 0, (D) $\ln(\frac{1}{2})$, (E) $e^{-\frac{1}{2}}$

3. Let $f(x) = \begin{cases} x^2 \sin(1/x) & x \neq 0 \\ 0, & x = 0 \end{cases}$ Then

   I. f(x) is differentiable at x=0.
   II. f′(x) is continuous at x=0.

   Which of the above two statements is (or are) true?
   (A) I, (B) II, (C) I and II, (D) none of them, (E) indecisive.

4. $\lim_{n\to\infty}(1+n+n^2)^{\frac{1}{n}} = ?$ (A) e, (B) e^2, (C) 1, (D) 0, (E) ∞.

5. $\int_0^1 \frac{x-1}{\ln x}dx = ?$ (A) e, (B) ln(2), (C) e^2, (D) sin 2, (E) cos 2.

6. I. $\sum_1^{\infty}\frac{n}{n^2+1}$, II. $\sum_2^{\infty}\frac{1}{n\ln n}$, III. $\sum_1^{\infty} ne^{-n^2}$

   Which of the above series converges? (A) I and II, (B) I and III, (C) II and III, (D) I, II, and III, (E) none of them.

7. If $f(x) = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \frac{x^6}{6!} + \cdots$, $x^2 < \infty$, then, f(x)=?

   (A) cos x, (B) $\frac{1}{2}(e^x + e^{-x})$, (C) sin x, (D) $\frac{1}{2}(e^x - e^{-x})$, (E) ln(1+x).

8. Find the present value of a perpetual income stream derived at a continuous rate of $100 + 3000e^{-t/2}$ dollars per year, which is discounted at a continuous interest rate of 10%. Which one of the following values is correct?
   (A) \$3100, (B) \$5000, (C) \$6000, (D) \$8450, (E) ∞.

9. f(x,y)= xy. The origin is
   (A) a saddle point, (B) a local minimum, (C) a local maximum, (D) an absolute minimum, (E) an absolute maximum.

10. Consider the following statements regarding matrix operations:

    I. The squareness is a sufficient condition for the existence of an inverse of matrix **A**.

    II. For matrixes **A** and **B**, if **AB=0**, then the only conclusions are ① either **A=0** or **B=0**, or ② both **A** and **B** are zeros.

    III. For matrixes **C**, **D**, and **E**, if **CD=CE**, **C**$\neq$**0**, then **D=E**.

    Which of the above three statements is (or are) true?

    (A) I and II, (B) II and III, (C) I and III, (D) I, II, and III, (E) none of them.

11. Let $(x+1)\frac{dy}{dx}=2y$. The solution to this differential equation is

    (A) y=ln(1+x) + c, (B) y = x+1+c, (C) $y=(x+1)^2+c$, (D) $y=c(x+1)^2$, (E) $y=ce^{(x+1)}$

12. A tennis club will charge an annual membership fee of $200 per member if 100 members or less joint. For each additional member, the fee is reduced by 50 cents. The club will admit at most 300 members. What is the size of membership that maximizes revenue for the tennis club?

    (A) 300, (B) 250, (C) 200, (D) 150, (E) 100.

貳、應用題: 共40分。

1. Assume that the utility function of a person for hamburgers (Y) and soft drinks (X) is $U(X,Y)=\sqrt{XY}$. Assume that hamburgers cost $1 each, soft drinks cost $0.25, and that this person has $2 to spend.

   (A) Find the best combination of hamburgers and soft drinks which will maximizes his personal utility. (4分)

   (B) Please check the second order condition for maximization. (3分)

   (C) How much utility will be raised for an increase in income of $1? (3分)

2. The moment generating function of a random variable X is defined as $M(t)=E(e^{tX})$, where E is the operator of expectation. Using the fact that the *m*th moment of distribution of the random variable $E(X^m)=M^{(m)}(0)$, calculate the mean (μ) and variance (σ^2) of the following gamma distribution

$$f(x)=\frac{1}{\Gamma(\alpha)\beta^{\alpha}}x^{\alpha-1}e^{-x/\beta}, \quad 0<x<\infty \qquad (15分)$$

$$=0, \quad \text{elsewhere}$$

3. (A) Prove that $\int_0^{\infty} e^{-x^2}dx=\frac{\sqrt{\pi}}{2}$ (10分)

   (B) Evaluate $\int_0^{\infty}\frac{1}{\sqrt{2\pi}}e^{-x^2/2}dx$ (5分)