注意：未寫明演算過程者不予計分。

(20%) (一) State which of the following are true and which are false. If false, please correct it.(若答案爲錯，則請更正)

1. If f is not continuous at a, then f is not differentiable at a.
2. If $f(x)>0, \forall x \in (a-\delta, a+\delta), \delta > 0$, and $\lim_{x\to a} f(x) = L$, then $L>0$.
3. If $f'(x)f''(x)<0$ for all x, then f is either increasing and concave down or decreasing and concave up.
4. If $g(t)=\int_0^{t^2} \sin u^2\, du$, then $g'(t)=\sin t^4$.
5. In polar coordinate system, the slope of the equation $r=f(\theta)$ at a point (r,θ) on the graph is $f'(\theta)$.
6. If $\{a_n\}$ is a monotone increasing sequence of positive numbers, then the sequence $\{a_n\}$ converges.
7. If $f_x(x_0,y_0)$ and $f_y(x_0,y_0)$ both exist, then f is continuous at (x_0,y_0).
8. If f is continuous at (x_0,y_0), and $f_x(x_0,y_0)$ and $f_y(x_0,y_0)$ both exist, then f is differentiable at (x_0,y_0).
9. If $\nabla f(x_0,y_0)=0$, then f has either a local maximum or a local minimum at (x_0,y_0).
10. If f is defined on the rectangular region $R=\{(x,y): a\le x\le b,\ c\le y\le d\}$, then $\iint_R f(x,y)\,dA=\int_a^b\int_c^d f(x,y)\,dy\,dx$.

(60%) (二) Multiple choice (Show all your work 請詳列演算過程，否則不予計分)

1. If $xy^2+x^2y=2$, find the value of $\frac{dy}{dx}$ at the point (1, 1).

A) 5	B) -1	C) -3	D) 1
E) -5	F) 0	G) 3	H) 4

2. Find the area under the Parabola $y=x^2$ from 0 to 3 can be done by using the limit of a sum of the form $\lim_{n\to\infty} \frac{c}{n^3}\sum_{i=1}^{n} i^2$. What is the value of the number c ?

A) 2	B) 4	C) 8	D) 27
E) 1/6	F) 1/4	G) 1/3	H) 1/2

(背面仍有題目,請繼續作答)

3. If $F(x)=\int_0^{\sqrt{x}} \frac{1}{\sqrt{1+t^4}}dt$, find the value of $F'(1)$.

A) $1/2$ B) $\sqrt{2}/2$ C) $\sqrt{2}/4$ D) $1/4$
E) $\sqrt{2}/8$ F) $1/8$ G) $\sqrt{2}/16$ H) $1/16$.

4. Find the are length of the graph of $(x+3)^2=8(y-1)^3$ from the point A(-2,3/2) to the point B(5,3).

A) $37^{3/2}$ B) $10^{3/2}$ C) $37^{3/2}-10^{3/2}$ D) $\frac{37^{3/2}}{27}$

E) none of these

5. If $f(x)=\ln(\sqrt{e^{2x}+e^{-2x}})$, then $f'(1)$ equals

A) 0 B) $\frac{e^2-e^{-2}}{\sqrt{e+e^{-2}}}$ C) $1/2\cdot\frac{e^2-e^{-2}}{\sqrt{e^2+e^{-2}}}$ D) $\frac{e^2-e^{-2}}{\sqrt{e^2+e^{-2}}}$

E) none of these

6. Find the value of the limit $\lim_{n\to\infty}(1+\frac{3}{x}+\frac{5}{x^2})^x$.

A) 0 B) 1 C) 3 D) 5
E) e^3 F) e^5 G) ln3 H) ln5

7. Find the value of the integral. $\int_0^1 \frac{\tan^{-1}x}{1+x^2}dx$

A) $\pi^2/32$ B) $\pi^2/24$ C) $\pi^2/16$ D) $\pi^2/12$
E) $\pi^2/8$ F) $\pi^2/4$ G) $\pi^2/2$ H) π^2

8. Find the value of the integral $\int_1^2 \frac{1}{x^3+x}dx$.

A) $3\ln 2-3$ B) $\ln 2-2\ln 3$ C) $\ln 2-\ln 5$
D) $(3\ln 2-\ln 5)/2$ E) $(3\ln 2-\ln 3)/2$ F) $(\ln 2-\ln 5)/2$

9. Which of the following series diverge?

(1) $\sum_{n=1}^{\infty}\frac{n+2}{n^2+1}$ (2) $\sum_{n=1}^{\infty}\frac{n!}{2^n}$ (3) $\sum_{n=1}^{\infty}\left(\frac{2n-1}{n+3}\right)^n$

A) none B) 1 C) 2 D) 3
E) 1, 2 F) 1, 3 G) 2, 3 H) 1, 2, 3

10. Which of the following series are conditionally convergent ?

(1) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ (2) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ (3) $\sum_{n=1}^{\infty} \frac{\cos n}{2^n}$

A) none B) 1 C) 2 D) 3
E) 1, 2 F) 1, 3 G) 2, 3 H) 1, 2, 3

11. Find the minimum value the function $f(x, y) = x^2 + y^2$ subject to the constraint $xy = 2$.

A) ½ B) 1 C) ³⁄₂ D) 2
E) ⁵⁄₂ F) 3 G) ⁷⁄₂ H) 4

12. Find the volume under the paraboloid $z = 4x^2 + y^2$ above the triangle with vertices (0, 0, 0), (3, 0, 0), and (3, 1, 0).

A) 109/4 B)55/2 C) 111/4 D) 28
E) 113/4 F) 57/2 G) 115/4 H) 29

(10%) (三) Let R be the disk with its center at the origin and radius 1. Evaluate

1. $\iint_R \sqrt{x^2 + y^2}\, dA$.
2. $\iint_R x\sqrt{x^2 + y^2}\, dA$.
3. $\iint_R e^{x^2+y^2}\, dA$.

(10%) (四) Evaluate $\iint_R (6x - 3y)\, dA$ where R is the region bounded by

$2x - y = 1$, $2x - y = 3$, $x + y = 1$, $x + y = 2$.