$$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{1}{a^2}\frac{x}{\sqrt{x^2+a^2}}, \quad \int \frac{xdx}{(x^2+a^2)^{3/2}} = -\frac{1}{\sqrt{x^2+a^2}}.$$

1. A rock is tied to a cord, and the other end of the cord is held fixed. The rock is given an initial tangential velocity that causes it to rotate in a vertical circle. Prove that the tension in the cord at the lowest point exceeds that at the highest point by six times the weight of the rock. (8 %)

2. A proton moving with speed v_{A1} in the $+x$-direction makes an elastic, off-center collision with an identical proton originally at rest. After impact the first proton moves with speed v_{A2} in the first quadrant at an angle α with the x-axis, and the second moves with speed v_{B2} in the fourth quadrant at an angle β with the x-axis. Prove that $\alpha + \beta = \pi/2$. (10 %)

3. The boom(吊桿) in Fig. 1 weighs 2600 N. The boom is attached with a frictionless pivot at its lower end. It is not uniform; the distance of its center of gravity from the pivot is 35% of its length, a) Find the tension in the guy wire (鋼索) and the horizontal and vertical components of the force exerted on the boom at its lower end. b) Does the line of action of the force exerted on the boom at its lower end lie along the boom? (8 %)

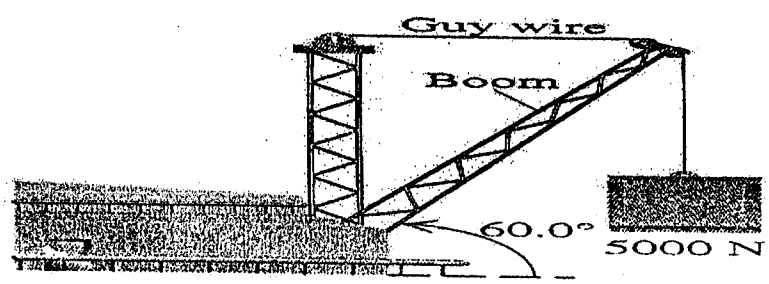

Fig. 1.

4. Consider a spring that exerts the following restoring force:

$$F = \begin{cases} -kx & \text{for } x > 0, \\ -2kx & \text{for } x < 0. \end{cases}$$

A mass m on a frictionless horizontal surface is attached to this spring, displaced to $x = A$ by stretching the spring, and released. a) Find the period of the motion. Does the period depend on A? Are the oscillations simple harmonic? b) What is the most negative value of x that the mass m reaches? Is the oscillation symmetric about the point $x = 0$? (8 %)

(背面仍有題目,請繼續作答)

5. You measure the period of a physical pendulum about one pivot point to be T. Then you find another pivot point on the opposite side of the center of mass that gives the same period. The two points are separated by a distance L. Use the parallel-axis theorem to find g (acceleration due to gravity) in terms of L and T. (10 %)

6. Many diatomic (two-atom) molecules are bound together by covalent bonds that are much stronger than the van der Waals interaction. Experiment shows that for many such molecules, the interaction can be described by a force of the form

$$F = A[\, e^{-2b(r-R_0)} - e^{-b(r-R_0)}],$$

where A and b are positive constants, r is the center-to-center separation of the atoms, and R_0 is the equilibrium separation. For the hydrogen molecule (H_2), $A = 2.97\times10^{-8}$ N, $b = 1.95\times10^{10}$ m^{-1}, and $R_0 = 7.4\times10^{-11}$ m. Find the force constant and frequency for small oscillations around equilibrium. (8 %)

7. An incompressible fluid with density ρ is in a horizontal test tube of inner cross- section area A. The test tube spins in a horizontal circle in an ultracentrifuge at an angular speed ω. Gravitational forces are negligible. Consider a volume element of the fluid of area A and thickness dr' a distance r' from the rotation axis. The pressure on its inner surface is p and on its outer surface is $p + dp$. a) Apply Newton's second law to the volume element to find dp. b) If the surface of the fluid is at a radius r_0 where the pressure is p_0, show that the pressure p at a distance $r > r_0$ is $p = p_0 + \rho\omega^2(r^2 - r_0^2)/2$. c) An object of volume V and density ρ_{ob} has its center of mass at a distance R_{cmob} from the axis. Find the net horizontal force on the object in terms of ρ, V, ω, and R_{cm}, where R_{cm} is the distance from the axis to the center of mass of the displaced fluid. (12 %)

8. a) With what speed is a solid brass sphere 2.50 mm in radius falling in a tank of glycerin at an instant when its acceleration is one half that of a freely falling body? The viscosity of the glycerin is 8.30 poise, b) What is the terminal speed of the sphere? ($\rho_{Cu} = 8.6\times10^3$ kg/m^3, $\rho_{glycerin} = 1.26\times10^3$ kg/m^3, 1 poise = 1 dyn·s/cm^2) (12 %)

9. Positive electric charge Q is distributed uniformly along a line with length $2a$, lying along the y-axis between $y = -a$ and $y = +a$, Find the electric field at point P on the x-axis at a distance x from the origin. (12 %)

10. In an L-R-C series circuit, the source has a voltage amplitude of 120 V, R = 80.0 Ω, and the reactance of the capacitor is 480 Ω. The voltage amplitude across the capacitor is 360 V. a) What is the current amplitude in the circuit? b) What is the impedance? c) What two values can the reactance of the inductor have? d) For which of the two values found in part (c) is the angular frequency less than the resonance angular frequency? Explain. (12 %)