

壹、選擇題 (請在下列各題中選出一個最佳的答案，每題 5 分) 60%

1. $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots + \frac{1}{(2n-1) \cdot (2n+1)} = ?$

- (A) 1, (B) $\frac{1}{2}$, (C) $\frac{1}{2n+1}$, (D) $\frac{n}{2n+1}$, (E) $\frac{n+1}{2n+1}$

2. $\lim_{x \rightarrow 0} \frac{3 \sin \pi x - \sin 3\pi x}{x^3} = ?$

- (A) 0, (B) π , (C) π^2 , (D) $2\pi^3$, (E) $4\pi^3$

3. Calculate dy/dx if $e^{xy} + y \ln x = \cos 2x$

- (A) $\frac{2x \sin 2x + xye^{xy} + y}{x^2 e^{xy} + \ln x}$, (B) $-\frac{2x \sin 2x + xye^{xy}}{x^2 e^{xy} + x \ln x}$, (C) $-\frac{2x \sin 2x + xye^{xy} + y}{x^2 e^{xy} + x \ln x}$,
(D) $\frac{2x \sin 2x + xye^{xy} + y}{xe^{xy} + y \ln x}$, (E) $-\frac{2x \sin 2x + xye^{xy} + y}{xe^{xy} + y \ln x}$

4. $\lim_{n \rightarrow \infty} \int_0^{2\pi} \frac{\sin nx}{x^2 + n^2} dx = ?$

- (A) 0, (B) 1, (C) -1, (D) π , (E) $-\pi$

5. $\int 3^{\sqrt{2x+1}} dx = ?$

- (A) $\frac{3^y}{\ln 3} - \frac{3^y}{(\ln 3)^2} + c$, (B) $\frac{y \cdot 3^y}{\ln 3} - \frac{3^y}{(\ln 3)^2} + c$, (C) $\frac{3^y}{\ln 3} - \frac{y \cdot 3^y}{(\ln 3)^2} + c$,
(D) $\frac{y \cdot 3^y}{(\ln 3)^2} - \frac{3^y}{\ln 3} + c$, (E) $3^{\sqrt{2x+1}}$

6. $\lim_{n \rightarrow \infty} \left\{ \frac{n}{n^2+1^2} + \frac{n}{n^2+2^2} + \dots + \frac{n}{n^2+n^2} \right\} = ?$

- (A) ∞ , (B) 1, (C) π , (D) $\frac{\pi}{2}$, (E) $\frac{\pi}{4}$

7. If $F(x, y) = x^3 y + e^{xy^2}$, find $F_{xy} = ?$

- (A) $4x^2 y^2 e^{xy^2} + 2xe^{xy^2}$, (B) $6xy + y^4 e^{xy^2}$, (C) $3x^2 + 2xy^3 e^{xy^2} + 2ye^{xy^2}$,
(D) $3x^2 + 2xy^2 e^{xy^2} + 2ye^{xy^2}$, (E) $3x^2 + 2xye^{xy^2} + 2y^2 e^{xy^2}$

8. If $U(x, y, z) = 2x^2 - yz + xz^2$, $x = 2 \sin t$, $y = t^2 - t + 1$, $z = 3e^{-t}$, find dU/dt at $t = 0$.

- (A) 24, (B) 20, (C) 16, (D) 8, (E) 4

(背面仍有題目, 請繼續作答)

9. What is the volume of the largest rectangular parallelepiped which can be inscribed in the ellipsoid $\frac{x^2}{9} + \frac{y^2}{16} + \frac{z^2}{36} = 1$?

- (A) $36\sqrt{3}$, (B) $64\sqrt{3}$, (C) $64\sqrt{2}$, (D) $128\sqrt{3}$, (E) $36\sqrt{2}$

10. If $F(\alpha) = \int_{\alpha}^{\alpha^2} \frac{\sin \alpha x}{x} dx$, find $F'(\alpha)$ where $\alpha \neq 0$

- (A) $\frac{3 \sin \alpha^3 - 2 \sin \alpha^2}{\alpha^2}$, (B) $\frac{\cos \alpha^3 - \cos \alpha^2}{\alpha}$, (C) $\frac{\sin \alpha^3 - \sin \alpha^2}{\alpha}$,
(D) $\frac{3 \sin \alpha^3 - 2 \sin \alpha^2}{\alpha}$, (E) $\frac{2 \sin \alpha^3 - 3 \sin \alpha^2}{\alpha^2}$

11. Which of the following series converges?

I. $\sum_1^{\infty} \frac{n}{n^2+1}$, II. $\sum_2^{\infty} \frac{1}{n \ln n}$, III. $\sum_1^{\infty} n e^{-n^2}$

- (A) I, (B) II, (C) III, (D) I & II, (E) I & III

12. If m and n are real numbers, then the integral $\int_0^1 x^{m-1} (1-x)^{n-1} dx$

- (A) converges for all $m, n \in R$.
(B) converges if $m > 1, n > 1$, and diverges otherwise.
(C) converges if $m < 1, n < 1$, and diverges otherwise.
(D) converges if $m < 0, n < 0$, and diverges otherwise.
(E) converges if $m > 0, n > 0$, and diverges otherwise.

貳、應用題: 40%

1. A rectangular box, open at the top, is to have a volume of 32 cubic feet. What must be the dimensions so that the total surface is a minimum?
(12 分)
2. A particle is attracted toward a fixed point O with a force inversely proportional to its instantaneous distance from O. If the particle is released from rest, find the time for it to reach O. (12 分)

3. A firm has purchased a machine that will produce additional revenue at a rate of

$$R'(x) = 625 + \frac{2500}{t+1}$$

dollars per year t years hence. The total maintenance and repair costs for the machine increase at a rate of

$$M'(x) = 625 + 100t$$

dollars per year t years hence. Suppose that the firm decides to sell the machine when incremental repair and maintenance costs exceed 80% of incoming revenue.

- (a) When should the machine be sold?
(b) Compute the total net earnings for this period and interpret this sum as an area.

(16 分)