I. Multiple choice (5 points each for #1~#8; 10 points each for #9 and #10)

1. $\lim_{n\to\infty}(\sqrt{n^{200}+n^{100}+1}-n^{100}) =$

(a) ∞ (b) 0 (c) 1/2 (d) 1 (e) $-\infty$

2. Define a sequence by $a_n = \int_0^1 (1-x^2)^n dx$, then $\lim_{n\to\infty}(\sqrt{a_n})^{1/n}$ is

(a) ∞ (b) 0 (c) 1 (d) -1 (e) 1/2

3. $\lim_{x\to 0} e^{-1/x^2} / x^4 =$

(a) ∞ (b) 0 (c) -1 (d) e (e) 1

4. If $|f(x)/x| \le M$ for $x \ne 0$, then $\lim_{x\to 0} f(x) =$

(a) ∞ (b) 0 (c) $M/2$ (d) 1 (e) M

5. Which statement is correct?

(a) If $\lim_{x\to a} f(x)g(x)$ exists then $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ both exist.

(b) If $\lim_{x\to a} |f(x)|$ exists then $\lim_{x\to a} f(x)$ exists too.

(c) If $\lim_{x\to a} f(x) = L$ then $\lim_{x\to a} |f(x)| = |L|$

(d) If $\lim_{x\to a} f(x)$ exists then $\lim_{x\to a} |f(x)|$ may not exist.

(e) $\lim_{x\to a} \frac{f(x)}{|f(x)|} = 1$

6. For the function $f(x, y) = x^3 + y^3 - 3xy$, which statement is correct?

(a) the partial derivative of $f(x, y)$ with respective to y is $3y^2$

(b) $f(x, y)$ has local minimum at (1,1)

(c) $f(x, y)$ has local maximum at (0,0)

(d) (1, 1) is a saddle point

(e) (1, 0) is a saddle point

7. $\int_0^4 min\{6x, 5+x^2\}dx =$

(a) 48 (b) 124/3 (c) 16/3 (d) 39 (e) 36

8. $\int_3^5 \frac{x-6}{x^2-2x} dx =$

(a) 3ln5−5ln3 (b) 5ln3−3ln5 (c) 5ln5-3ln3 (d) ln3 (e) ln5

(背面仍有題目,請繼續作答)

9. Select incorrect statements. (there may be more than one incorrect statements)

(a) $\frac{x}{1+x} \le \ln(x+1) \le x$ for $x > -1$ and $x \ne 0$

(b) $\ln(x+1) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{n+1}}{n+1}$

(c) $\int_1^3 \ln(x+1)dx = -(1/4)$

(d) $\lim_{x \to 0} x \ln(1+x) = 0$

(e) The integral $\int_0^{\infty} (\ln(x+1))/(x+1)dx$ converges

10. Select correct statements. (there may be more than one correct statements)

(a) If $f(x)$ is continuous at $x=5$ then $\lim_{x \to 5} f(x) = f(5)$.

(b) If $f'(x) = g'(x)$ then $f(x) = g(x)$.

(c) The greatest integer function is denoted by $[x]$ which represents the greatest integer <= x, and it is continuous at every real number x.

(d) Given that $|f'(x)| \le 1$ for all real numbers x, then $|f(z) - f(y)| < |z-y|$.

(f) If $f(x)$ is differentiable in $[a, b]$ then there is at least one c in $[a, b]$ such that

$$\int_a^b f(x)dx = f(c)(b-a)$$

II. Applications and Computation (Please show all work and 10 point for each problem)

1. A manufacturer can produce three distinct products in quantities x, y, z, respectively, and thereby derive a profit $f(x, y, z) = 2x + 8y + 24z$. Find the values of x, y, z that maximize profit if production is subject to the constraint $x^2 + 2y^2 + 4z^2 = 4.5\text{x}10^9$.

2. Show that $n\binom{n-1}{b-1}\int_0^p x^{b-1}(1-x)^{n-b}dx$ is equal to $\sum_{k=b}^{n} \binom{n}{k} p^k (1-p)^{n-k}$

3. Would you rather collect NT15,000 per year for 10 years or NT10,000 per year in perpetuity? Assume that banks pay an annual interest of 10% compounded continuously.

4. Find $\iint_R \frac{1}{y} dx\, dy$, where R is the region bounded by the curves $y^3 = x^2$, $y^3 = 4x^2$, and the lines $y = x$, $y = 5x$.