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I.　Multiple choice ( 5 points for each problem)

1.　$\lim\limits_{h \to 0} \dfrac{\sqrt{h+1}-1}{h} =$

(a)　0　　(b)　$\dfrac{1}{\sqrt{h+1}+1}$　　　(c)　$\dfrac{1}{\sqrt{h+1}-1}$　　　(d)　$\sqrt{h+1}+1$

(e)　$\sqrt{h+1}-1$

2.　$\lim\limits_{x \to 4} \dfrac{x^2-16}{4\sqrt{x}-8} =$

(a)　0　　(b)　2　　(c)　4　　(d)　8　　(e)　16

3.　$\displaystyle\int_1^e \dfrac{\ln y}{y}\,dy =$

(a)　2　　(b)　$\dfrac{1}{2}$　　(c)　3　　(d)　$\dfrac{1}{3}$　　(e)　$\dfrac{1}{4}$

4.　$\displaystyle\int_1^3 \dfrac{1}{x^2} e^{2/x}\,dx =$

(a)　$\dfrac{1}{2}\left(e^2 - e^{2/3}\right)$　　(b)　$2\left(e^2 - e^{2/3}\right)$　　(c)　$\dfrac{1}{4}\left(e^2 - e^{1/2}\right)$　　(d)　$4\left(e^2 - e^{1/2}\right)$

(e)　$\dfrac{1}{2}\left(e^2 - e^{1/2}\right)$

5.　Find　$F'(t)$　in the following case:

$$F(t) = \int_t^{2t} x^2\,dx$$

(a)　$3t$　　(b)　$7t$　　(c)　$t$　　(d)　$7t^2$　　(e)　$3t^2$

6.　$\lim\limits_{x \to 1} \dfrac{x^2+3x-4}{2x^2-2x} =$

(a)　$\dfrac{1}{2}$　　(b)　$\dfrac{3}{2}$　　(c)　$\dfrac{5}{2}$　　(d)　$\dfrac{7}{2}$　　(e)　3

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II.　Applications and Computation (Please show all work and 10 points for each problem)

1.　A curve in the $uv$-plane is given by

$$u^2 + uv - v^3 = 0$$

Compute $dv/du$ by implicit differentiation. Find the point $(u,v)$ on the curve where $dv/du = 0$ and $u \neq 0$.

2.　Establish the approximation

$$\left(1 + \frac{p}{100}\right)^n \approx 1 + n\frac{p}{100} + \frac{n(n-1)}{2}\left(\frac{p}{100}\right)^2$$

3.　Explain why the function $f$ defined on $[0, \sqrt{3}]$ by

$$f(x) = \frac{1}{3}x^3 \sqrt{4-x^2}$$

has an inverse function $g$. Compute $g'(\frac{1}{3}\sqrt{3})$. (*Hint:* $f(1) = \frac{1}{3}\sqrt{3}$)

4.　Consider the function $f$ defined for all $x$ by $f(x) = xe^x$.
(a)　Compute $f'(x)$ and $f''(x)$. Find the intervals on which $f$ is increasing.
(b)　Draw the graph of $f$.

5.　Are the following functions concave/convex? (assuming that $x > 0$ in part (a))
(a)　$2x - 3 + 4\ln x$
(b)　$3x^2 - 2x + 1 + e^{-x-3}$

6.　The profit of a firm as a function of its output $x$　$(x > 0)$　is

$$f(x) = 4000 - x - \frac{3000000}{x}$$

(a)　Find the output that maximizes profit. Draw the graph of $f$.
(b)　The actual output varies between 1000 and 3000 units. Compute the average profit

$$I = \frac{1}{2000} \int_{1000}^{3000} f(x)\,dx$$

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7. Solve the following problem by Lagrangean method. Prove that you have found the optimal solution.

$$\text{Max} \quad x^2 + 3xy + y^2 \quad \text{subject to} \quad x + y = 100$$

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