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I. True or False (30 points, 3 points each)

1. A sequence function of $f_n(x) = \dfrac{x}{n+x^2}$ on [0,1] converges uniformly.

2. All continuous functions have derivatives.

3. The sequence $b_n = 0.15^n$ converges to $\dfrac{1}{1-0.15}$.

4. If $\lim_{n\to\infty} a_n = 0$, then $\sum_{n}^{\infty} a_n$ can be said to converge.

5. The domain of function $f(x) = (x+1)^x$ is the interval of $(-1, \infty)$

6. If the trajectory of $f(x)$ runs through the points (0,0) and (2,2) and $f'(x)$ is continuous, then $\int_0^2 f'(x)\,dx = 2$.

7. $f(x) = |x|$ has an antiderivative on the interval [-1,1].

8. Assuming f is any odd function and continuous on [-c, c], where c is a constant. Then, the area between $y = f(x)$, -c, c and $y = 0$ is zero.

9. Any polynomial function is continuous.

10. If $f(x)$ and $g(x)$ defined in R are uniformly continuous, then $f(x)g(x)$ is also uniformly continuous.

II. Multiple Choice Questions (40 points, 4 points each, Only one choice)

1.　　　　Let $f(x) = e^x$ be defined on the interval [0,4]. What is the average value of f?

(A) $\dfrac{e^4 - 1}{4}$,

(B) $\dfrac{e^4 - 1}{2}$,

(C) $\dfrac{e^2 - 1}{2}$.

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2.   What is the value, c, making the average value of f equal to $f(c)$?

     (A) 1.1614,

     (B) 3.2884,

     (C) 2.5952.

3.   The velocity function is $v(t) = 3t - 7$, $0 \leq t \leq 4$, what is the displacement?

     (A) $\dfrac{7}{3}$,

     (B) -4,

     (C) -7.

4.   Based on question 3 above, what is the distance traveled?

     (A) $\dfrac{65}{4}$,

     (B) 5,

     (C) $\dfrac{31}{2}$.

5.   The demand for a certain sedan model is 12,000 units when the price is NT$500,000. The demand increases by 1,000 units if the price is drops by NT$20,000. Therefore, the demand function is

     (A) $-\dfrac{1}{20} price + 2,000$,

     (B) $-\dfrac{1}{20} price + 37,000$,

     (C) $-20 price + 10,012,000$.

6.   Based on question 5 above, what are the demand units once the price is NT$450,000?

     (A)150,000 units,

     (B)10,012,000 units,

     (C) 14,500 units.

系所組別： 企業管理學系丁組
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7.　　What is the natural domain of $f(x) = \dfrac{x-2}{x^3 - 14x^2 + 59x - 70}$?

(A) natural domain is where $x \neq 0,3,5$,

(B) natural domain is where $x \neq 2,5,7$,

(C) natural domain is where $x \neq 0,2,7$.

8.　　Based on the question 7 above, what are the vertical asymptotes of the function f?

(A) $x=5,7$,

(B) $x=0,5$,

(C) $x=2,7$.

9.　　What is the y in terms of x for the following initial value problem:

$$\frac{dy}{dx} = 3y(x+1), \text{ and } y(0) = e^9?$$

(A) $y = \exp\left(\frac{9}{2}x^2 + 3x + 1\right)$,

(B) $y = \exp\left(\frac{1}{2}x^2 + 9x + 1\right)$,

(C) $y = \exp\left(\frac{3}{2}x^2 + 3x + 9\right)$.

10.　　Based on question 9, what is the value of y when $x=2$?

(A) e^{20},

(B) e^{21},

(C) e^{22}.

（背面仍有題目，請繼續作答）

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III. Partial Credit Questions (30 points)

1. Let

$$f(x, y) = 3x^3 + 2y^3 - 3x + 4y + 12$$

(a) Find the critical points of $f(x, y)$. (6 points)

(b) Find which critical points are the relative maximum and relative minimum

points of $f(x, y)$. (4 points)

(c) Find which critical points are the saddle points of $f(x, y)$. (4 points)

2. Let

$$y' = x^2 - y, \text{ and } y(0) = 1.$$

(a) Using third-order Taylor expansion to solve y in terms of x. (10 points)

(b) Approximate $y(2)$ using the third-order Taylor expansion. (6 points)