※ 考生請注意：本試題 ☑可 □不可 使用計算機

Please show all your work.

1. (10%) Two people agree to meet at one place "sometime around 7:30am". Neither of them is particularly punctual or patient. What will actually happen is that each will arrive at random sometime in the interval from 7:00am to 8:00am. If one arrives and the other is not there, the first person will wait 15 minutes or until 8:00am, whichever comes first, and then leave. What is the probability that the two will meet?

2. (10%) Suppose that two continuous random variables X and Y are jointly distributed. Their joint probability density function is $f_{X,Y}(x,y) = 8xy$, $0 \le x < y < 1$. What is their covariance?

3. (10%) Please explain (1) Type I error; (2) the power of test. (5% for each question)

4. (10%) Please show your understanding of (1) degrees of freedom; (2) the Student's t distribution. (5% for each question)

5. (10%) (1) How do you find probabilities from a normal distribution? (2) Describe the relation between normal distributions and the standard normal distribution. (5% for each question)

6. (10%) (1) Please define multicollinearity. (2) How to detect multicollinearity in a sample of data? (5% for each question)

7. (10%) Please explain the following terms: (1) a standard deviation; (2) a best linear unbiased estimator of a parameter. (5% for each question)

8. (10%) Assume that X is a continuous random variable and follows the density function $f_X(x) = 2x$, $0 < x < 1$. Define $g(X) = 3X^2 - 1$. Please find the variance of $g(X)$.

9. (10%) Let Y follow a uniform distribution in the interval (0, 1). Find the probability density function of $X = Y^3$.

10. (10%) The binomial distribution is a distribution of the number of successes in n independent trials where the probability of success in each trial is p. This distribution is represented by $B(n, p)$. Let X have the $B(20,0.2)$ distribution. Find a value of n such that $P(X > 0) = 0.97$.