※ 考生請注意：本試題不可使用計算機　　請勿在本試題紙上作答，否則不予計分

一、True or False 2%× 15=30%

For the following statements, please answer **T** *if it is true and* **F** *otherwise.*

1. The sample mean of a simple random sample is an unbiased estimator of the population mean for any population.

2. X and Y are two random variables with respective expected values μ_x and μ_y, and $U = X + Y$, then the equality $\mathbf{E}(U) = \mu_x + \mu_y$ requires the assumption that X and Y are independent.

3. If two events A and B are independent, then they are disjoint as well.

4. If **S** is a sufficient statistic, and **T** is a function of **S** (i.e. **T** can be computed from **S**), then **T** is a sufficient statistics as well.

5. The two-sample t test is a not a uniformly most powerful (UMP) test to examine $H_o : \mu = \mu_o$ against $H_a : \mu \neq \mu_0$ even if all the assumptions are satisfied.

6. If there are two sequence of random variables X_n and Y_n and $X_n \xrightarrow{p} a$, $Y_n \xrightarrow{p} b$, then $g(X_n, Y_n) \xrightarrow{p} g(a, b)$ as long as $g(x, y)$ is continuous at (a, b).

7. If a random variable $X \geq a$, then $\mathrm{E}X \geq a$.

8. If $\hat{\theta}$ is the maximum likelihood estimator (MLE) for θ and it is unbiased ($\mathrm{E}\hat{\theta} = \theta$), then $\hat{\theta}$ is the best unbiased estimator.

9. If $\mathbf{X}_n = (X_{1n}, \ldots, X_{mn})'$ is a sequence of random vectors, then $\mathbf{X}_n \xrightarrow{p} \mathbf{a} = (a_1, \ldots, a_m)'$ assures that $X_{in} \xrightarrow{p} a_i, \forall i = 1, \ldots, m$, but the reverse is not necessarily true.

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10. If $X_i \overset{iid}{\sim} N(\mu, \sigma^2)$, then $T = n^{1/2}(\bar{X} - \mu)/\sigma$ is a pivotal quantity for μ since the distribution of T is completely known as $T \sim N(0,1)$.

11. If T_n is an unbiased estimator of θ, where T_n is a statistic base on $X_1, \ldots, X_n$, then T_n is also consistent for θ as long as $\mathrm{Var}(T_n) \to 0$.

12. $X_1, \ldots, X_k$ are independently distributed Exponential random variables, then $T = \sum_{i=1}^{k} X_i$ is distributed as a Gamma distribution.

13. If X and Y are independently and identically distributed as standard normal distribution and $W = X/Y$, then $\mathrm{E}W = 1$ and $\mathrm{Var}(W)$ does not exist.

14. If X and Y are independent random variables with moment generating function $M_X(t) = (1-t)^{-2}$ and $M_Y(t) = (1-t)^{-3}$, then the moment generating function of $V = X + Y$ has moment generating function $M_V(t) = (1-t)^{-5}$.

15. If both of the conditional distribution of X given Y and the distribution of Y are normal, then the marginal distribution of X is a normal one as well.

二、 Fill in the Blanks 4% × 5=20%

1. Let Y be uniformly distributed on $(-\theta, \theta)$, and the conditional distribution of X given $Y = y$ is a uniform one as

$$(X|Y = y) \sim \begin{cases} Uni(0, y), & \text{if } y \geq 0 \\ Uni(y, 0), & \text{if } y < 0 \end{cases}$$

, then $\mathrm{E}X =$ ____1____ and $\mathrm{Cov}(X, Y) =$ ____2____

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2. If X and Y are jointly distributed as a trinomial distribution with parameters n, θ^2, $2\theta(1-\theta)$, and $(1-\theta)^2$, then the best unbiased estimator of θ is____3____

3. There is a coin which we know if it is not a fair one, then the probability of head is 0.8. If it is flipped and the the result is head, what would you do you for the most powerful size-0.1 test to examine

$$H_o: \text{ It is a fair coin.}$$

against

$$H_a: \text{ It is not a fair coin}$$

____4____

4. Let $\bar{X}_n$ be the sample mean computed from a sample with mean μ and variance $\sigma^2 < \infty$, then the asymptotic distribution of $U_n = n^{1/2}(\bar{X}_n^2 - \mu^2)$ is $U_n \xrightarrow{d}$ ____5____

三、Problems 50%

1. (10%) Consider a game in which the player is going to flip a fair coin until he/she gets a head. Also, if the head comes on the k_{th} toss, the player gets NTD 2^k. What is your expected return in this game, and how much would you like to pay to play this game?

2. (10%) Let $\mathbf{X}$ be a continuous random vector with variance-covariance matrix $\mathbf{\Sigma}$, please prove that $\mathbf{\Sigma}$ is positive definite.

3. (15%) Let $X_1, \ldots, X_n$ be random sample from a Poisson distribution with mean θ. Is there an efficient unbiased estimator for θ^2? Find it if there is one, and give the reason if it does not exist.

4. (15%) Let $X_1, \ldots, X_n$ be random sample from $N(\theta, \theta^2)$, is there a complete sufficient statistic for this model? Find it if there is one, and give the reason if it does not exist.