國立成功大學105學年度碩士班招生考試試題

系　　所：統計學系

考試科目：數學

考試日期：0228，節次：1

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Let $f(x) = e^{|x-1|}$. (10%)

   (a) Is it differentiable at $x = 1$?

   (b) What is the near approximation of $f(x)$ near $a = 0$?

2. Find the following limits. (20%)

   (a) $\lim_{h\to 0} \ln\sqrt{(1-2h)^{3/h}}$　　(b) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{k}{3^k}$

   (c) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{(-3)^k}{k!}$　　(d) $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{k3^k}$

3. Find the following integrals. (20%)

   (a) $\int_0^{\infty} x^2 e^{-x^2}\,dx$　　(b) $\int_0^{0.5} x^2\sqrt{(1-4x^2)}\,dx$

   (c) $\int_0^1 x/\sqrt{1-x^2}\,dx$　　(d) $\int_{-\infty}^{\infty} e^{-x^2}\,dx$

4. Find the following derivatives. (10%)

   (a) $\frac{d}{dx}\ln|\sin x^2|$ at $x = \sqrt{3\pi/2}$　　(b) $D_t \cos^{-1}(1-t^2)$ at $t = 0$

5. Let $f(x) = \int_{\sqrt{x}}^{x} \ln(t^2)\,dt$ and $g(x) = e^{2x}$. Find the slope of the tangent line passing through $y = f(g(x))$ at $x = 1$. (10%)

6. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 3 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$. (10%)

   (a) Find a basis for the column space of A. (5%)

   (b) If the vector b is the sum of the four columns of A, write down the complete solution (also called the general solution) to $Ax = b$. (5%)

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7. Let $A=\begin{bmatrix}1 & 2\\ 2 & 4\\ 3 & 6\end{bmatrix}$. (10%)

(a) Find a complete set of orthonormal eigenvectors of AA^T. (5%)

(b) If A is any m by n matrix with $m>n$. Is it possible that AA^T is positive definite? Is A^TA always positive definite? Why? (5%)

8. Suppose A and B are n by n matrices, and $AB=I$. (10%)

(a) What is rank(A)? Why? (5%)

(b) Is A invertible? (3%)

(c) Is $BA=I$? (2%)