※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

Please show ALL of your works.

1. Let f and g be two functions defined as follows:

$$f(x) = \begin{cases} 1, & \text{if } |x| \le 1; \\ 0, & \text{if } |x| > 1, \end{cases} \quad g(x) = \begin{cases} 2 - x^2, & \text{if } |x| \le 2; \\ 2, & \text{if } |x| > 2. \end{cases}$$

Find a formula for computing the composite function $h(x) = f[g(x)]$. (6%) For what values of x is h continuous? (4%)

2. Evaluate the following limits. (10%)

   (a) $\lim_{x \to 0} \frac{(2-x)e^x - x - 2}{x^3}$ (5%)　(b) $\lim_{x \to 0} \frac{x^x - x}{1 - x - \log x}$ (5%)

3. Evaluate the following integrals. (20%)

   (a) $\int t(t+1)^{1/4} dt$ (5%)　(b) $\int \sin^3 x \, dx$ (5%)
   (c) $\int \frac{2x^2 + 5x - 1}{x^3 + x^2 - 2x} dx$ (5%)　(d) $\int \frac{1}{1+e^x} dx$ (5%)

4. Use integration by parts to derive the following recursion formula

$$\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx.$$

   (10%)

5. Find a function f, continuous for all x (and not everywhere zero), such that

$$f^2(x) = \int_0^x f(t) \frac{\sin t}{2 + \cos t} dt.$$

   (10%)

6. Let $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$. Verify that $A^2 = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$. (2%) Guess a general

   formula for A^n and prove it by induction. (8%)

7. Find the inverse of

$$\begin{pmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 2 \\ 3 & 4 & 2 & 5 \end{pmatrix}.$$

   (20%)

8. Find the maximum and minimum values of the ratio $\mathbf{x}^T A \mathbf{x} / (\mathbf{x}^T \mathbf{x})$ for any nonzero vectors $x^T = (x_1, x_2, x_3)$ if

$$A = \begin{pmatrix} 13 & -4 & 2 \\ -4 & 13 & -2 \\ 2 & -2 & 10 \end{pmatrix}.$$

   (10%)