國立成功大學

110學年度碩士班招生考試試題

編　號：239

系　所：統計學系

科　目：數學

日　期：0203

節　次：第 1 節

備　註：不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (15%, 5% for each) Evaluate the following integrals.

(a) $\int_1^9 e^{\sqrt{x}}dx$;　　(b) $\int_{-2}^2 \frac{3x^5+5x^3}{\sqrt{x^4+x^2}}dx$;　　(c) $\int_0^2 \int_x^2 e^{-y^2}dydx$.

2. (10%) Let $y = \frac{1}{1-x^2}$. Find the general formula for $y^{(n)}$ (the nth order derivative of y) for each positive integer n, and then prove by mathematical induction.

3. (15%) Find the equation of the tangent line to the function

$$y = \frac{x}{\sqrt{12+(x^2-9)\sqrt{\frac{x^2+3x+3}{\cos(x+1)}}}}$$

at the given point $x = -1$.

4. (a) (10%) Identify the quadratic function whose equation is $5x^2 + 5y^2 + 4xy - 21 = 0$ by rotating the xy-axes to put this quadratic function in the standard position. What is the angle through which you rotated the xy-axes?

(b) (10%) Find the values of x and y to maximize and minimize the following quadratic function

$$f(x,y) = 5x^2 + 5y^2 + 4xy,$$

subject to the constrain of $x^2 + y^2 = 1$.

(c) (5%) Explain the geometric meaning of (b) by drawing the figures.

5. (15%) Evaluate the eigenvalues and the corresponding eigenvectors of matrix A where

$$A = \begin{bmatrix} \theta & \rho & \rho & \rho & \rho \\ \rho & \theta & \rho & \rho & \rho \\ \rho & \rho & \theta & \rho & \rho \\ \rho & \rho & \rho & \theta & \rho \\ \rho & \rho & \rho & \rho & \theta \end{bmatrix},$$

with $\rho \neq 0$. Diagonalize A if possible.

6. (10%) Prove that if $\lim_{x \to c} g(x) = m$ and if f is continuous at m, then

$$\lim_{x \to c} f(g(x)) = f(m).$$

7. (10%) Find a subset of the given vectors that forms a basis for the space spanned by those vectors. Then express each vector that is not in the basis as a linear combination of the basis vectors.

$$\mathbf{v}_1 = (1,-1,5,2); \quad \mathbf{v}_2 = (-2,3,1,0); \quad \mathbf{v}_3 = (4,-5,9,4);$$
$$\mathbf{v}_4 = (0,4,2,-3); \quad \mathbf{v}_5 = (-7,18,2,-8);$$