國立成功大學

110學年度碩士班招生考試試題

編　號：240

系　所：統計學系

科　目：數理統計

日　期：0203

節　次：第 2 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Let $Y_1, \dots, Y_n$ be a random sample from the uniform distribution on the interval $(0, \theta)$ with an unknown $\theta \in (1, \infty)$. Suppose that we only observe

$$X_i = \begin{cases} Y_i & \text{if } Y_i \geq 1; \\ 1 & \text{if } Y_i < 1, \end{cases} \qquad i = 1, \dots, n.$$

   a) (10%) Derive a UMVUE of θ.

   b) (10%) Derive a UMP test of size $\alpha \in (0, 1)$ for testing $H_0: \theta \leq \theta_0$ versus $H_1: \theta > \theta_0$, where $\theta_0 > 1$ is known.

2. Let $U_1, U_2, \dots$ be independent random variables having the uniform distribution on $[0, 1]$. Find the limiting distributions of the following statistics Y_n and Z_n, respectively.

   a) (10%) $Y_n = (\prod_{i=1}^{n} U_i)^{-1/n}$.

   b) (10%) $Z_n = 2n(1 - R)$, where $R = U_{(n)} - U_{(1)}$ and $U_{(i)}$ is the ith order statistic.

3. If $X, X_n, n \geq 1$ are random variables such that $X_n \xrightarrow{P} X$ as $n \to \infty$.

   a) (10%) Let $g(x), x \in R$ be a continuous function. Show $g(X_n) \xrightarrow{P} g(X)$.

   b) (10%) Let

$$h(x) = \begin{cases} 0 & \text{if } x \leq 0; \\ 1 & \text{if } x > 0. \end{cases}$$

   Show that $h(X_n)$ does not converge in probability to $h(X)$.

4. Let $Y_1, Y_2, \dots, Y_n$ denote a random sample from a Poisson distribution with a mean λ. We assume that a prior for λ is a gamma distribution $\Gamma(\alpha, \beta)$ with the mean $\alpha\beta$.

   a) (10%) Show that the Bayes estimator (posterior mean) of λ is a weighted average of sample mean and the prior mean.

   b) (10%) Show that the Bayes estimator is a biased but consistent estimator of λ.

5. (20%) Consider a random variable X that has the logistic distribution with density function

$$f(x) = \frac{e^x}{(1+e^x)^2}, \quad -\infty < x < \infty.$$

Find the mean and variance of X.