

國立成功大學

111學年度碩士班招生考試試題

編 號： 234

系 所： 統計學系

科 目： 數學

日 期： 0220

節 次： 第 1 節

備 註： 不可使用計算機

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%, 5% for each) Calculate the sum.

(a). $\sum_{n=3}^{\infty} \frac{\ln(n)}{\ln(\ln(n))}$.

(b). $\sum_{n=1}^{\infty} \frac{x^n}{n}$. Find the series' radius and interval of convergence. Then identify the values of x for which the series converges absolutely and conditionally.

2. (15%, 5% for each) Calculate the integrations.

(a). $\int \sqrt{\frac{x-1}{x^5}} dx$. (b). $\int_0^{\pi} (1 - \sin(2x))^{\frac{3}{2}} \cos(2x) dx$. (c). $\int_0^1 \int_y^1 x^2 e^{xy} dx dy$.

3. (10%, 5% for each) Find the derivatives. (a). $y = \int_{\sqrt{x}}^{\sqrt[3]{x}} \log(t) dt$. (b). $y = \log_3 \left(\frac{x+1}{x-1} \right)^{\ln 3}$.

4. (10%) Prove or disprove the equation. $\lim_{a \rightarrow \infty} \int_{-a}^a \frac{2x dx}{x^2+1} = \int_{-\infty}^{\infty} \frac{2x dx}{x^2+1}$.

5. (10%, 5% for each)

(a) List the formula of Newton's method for estimation a solution of an equation $f(x) = 0$.

(b) Use Newton's method to estimate the two zeros of the function $f(x) = 2x - x^2 + 1$.

Start with $x_0 = 0$ for the left-hand zero and with $x_0 = 2$ for the zero on the right. Then, in each case, find x_2 .

6. (10%) A and B are two square matrices. Prove or disprove the equation

$$\det(A + B) = \det(A) + \det(B).$$

7. (10%, 5% for each) $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ and $\det(A) = -2$. Calculate the determinants.

(a). $\det((2A)^{-1})$. (b). $\det \left(2 \begin{bmatrix} a & g & d \\ b & h & e \\ c & i & f \end{bmatrix} \right)$.

8. (10%) For which values of α do the following vectors form a linearly independent set in R^3 ?

$$v_1 = \left(\alpha, -\frac{1}{2}, -\frac{1}{2} \right), v_2 = \left(-\frac{1}{2}, \alpha, -\frac{1}{2} \right), v_3 = \left(-\frac{1}{2}, -\frac{1}{2}, \alpha \right).$$

9. (5%) Let V be an inner product space. Show that if u and v are orthogonal unit vectors in V , then

$$\text{calculate } 10||u - v||.$$

10. (10%) Find the eigenvalues of the matrix A. $A = \begin{bmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$.