國立成功大學

112學年度碩士班招生考試試題

編　號：232

系　所：統計學系

科　目：數學

日　期：0207

節　次：第 1 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。 請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) Let Ω be the region between $y = x^2\cos(x^2)$ and the x-axis, $0 \le x \le \sqrt{\pi/2}$. Find the volume of the solid obtained by rotating Ω about the y-axis.

2. Set

$$h(x) = \int_0^x \frac{1}{1+u^2}du$$

(a) (5%) Show that $h(x)$ has an inverse.

(b) (5%) The inverse function of h is denoted as h^{-1}. Find the derivative of h^{-1} at $\pi/4$, $(h^{-1})'(\pi/4)$.

3. Find the limit, if exists, or show that the limit does not exist.

(a) (5%) $\lim_{n\to\infty}\left(\frac{4n-1}{4n+3}\right)^{n+1}$

(b) (5%) $\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{1}{n+\sqrt{kn}}\right)$

4. (10%) Set

$$f(x,y) = \frac{2xy}{x^2+y^2}$$

Show that $f(x,y)$ do not have a limit as $(x,y) \to (0,0)$.

5. (10%) Prove or disprove that

$$f(x) = \begin{cases} x^2\sin 1/x & ,x \neq 0 \\ 0 & ,x = 0 \end{cases}$$

is differentiable at $x = 0$.

6. (15%) For a square real matrix A, define $e^A = \lim_{m\to\infty}\sum_{k=0}^{m}\frac{1}{k!}A^k$, where A^0 is defined to be the identity matrix I. Evaluate e^A for

$$A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}.$$

Show your work.

7. (15%) Compute the QR decomposition (the QR factorization) of

$$A = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix}.$$

Show your work.

8. For each T, prove that T is a linear transformation, and find bases for the null space of T, $N(T)$, and the range of T, $R(T)$, respectively.

(a) (5%) $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x_1, x_2) = (x_1 + x_2, 0, x_1 - x_2)$

(b) (5%) $T: P_3(\mathbb{R}) \to P_2(\mathbb{R})$ defined by $T(f(x)) = f'(x) + x^2$.

(Note that $P_k(\mathbb{R})$ is the set of all polynomials degree less than or equal to k with coefficients from $\mathbb{R}$.)

9. (10%) Let

$$M = \begin{bmatrix} A & B \\ C & D \end{bmatrix},$$

in which A and D are square matrix and D is invertible. **Schur complement** of the block D of the matrix M is defined by $M/D \equiv A - BD^{-1}C$. You are given the result that

$$\det\left(\begin{bmatrix} A & B \\ \mathbf{0} & D \end{bmatrix}\right) = \det(A)\det(D),$$

where $\mathbf{0}$ is a zero matrix. Using this result to show $\det(M) = \det(D)\det(M/D)$.