國立成功大學

113學年度碩士班招生考試試題

編　號：226

系　所：統計學系

科　目：數理統計

日　期：0202

節　次：第 2 節

備　註：不可使用計算機

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (10%) Suppose that independent random sample of size n from two normal populations with the known variances σ_1^2 and σ_2^2 are to be used to test the null hypothesis $\delta = \mu_1 - \mu_2$ against the alternative hypothesis $\delta' = \mu_1 - \mu_2$ and that the probabilities of type I and type II errors are to have the preassigned value α and β, where $0 < \alpha < 1$ and $0 < \beta < 1$. Find the size of the sample to meet the requirement.

2. Let $Y_1, \ldots, Y_n$ be independent and identically distributed random variables with discrete probability function given by

	y		
	1	2	3
$p(y\|\theta)$	θ^2	$2\theta(1-\theta)$	$(1-\theta)^2$

where $0 < \theta < 1$. Let N_i denote the number of observations equal to i for $i = 1, 2, 3$.

a. (5%) Derive the likelihood function $L(\theta)$ as function of N_i, for $i = 1, 2, 3$.

b. (10%) Find the most powerful test for testing $H_0\colon \theta = \theta_0$ versus $H_1\colon \theta = \theta_1$, where $\theta_0 < \theta_1$. Show that your test specifies that H_0 be rejected for certain values of $2N_1 + N_2$.

c. (5%) How do you determine the value of k so that the test has nominal level α? You need not do the actual computation. A clear description of how to determine k is adequate.

d. (5%) Is the test derived in parts (a)-(c) uniformly most powerful for testing $H_0\colon \theta = \theta_0$ versus $H_1\colon \theta = \theta_1$? Why or why not?

3. Suppose that a random sample of length-of-life measurements, $Y_1, \ldots, Y_n$, is to be taken of components whose length of life has an exponential distribution with a mean θ. It is frequently of interest to estimate

$$\bar{F}(t) = 1 - F(t),$$

the reliability at time t of such a component.

a. (5%) For any fixed value of t, find the MLE of $\bar{F}(t)$.

b. (10%) Find the minimum-variance unbiased estimator of $\bar{F}(t)$.

4. Consider a random sample $Y_1, \ldots Y_n$ from a Poisson distribution with a mean θ. Suppose that the prior distribution of θ is from a gamma distribution, $\Gamma(\alpha, \beta)$, where α is the shape parameter and β the rate (inverse of scale) parameter and α and β are known.

a. (10%) Find the posterior mean of θ.

b. (10%) Find posterior predictive distribution, $p(\tilde{y}|y_1, \ldots y_n)$, where $\tilde{y}$ is the predictive value and $y_1, \ldots y_n$ are observed values.

5. Suppose that $X_1, \ldots, X_n$ are independent and identically distributed Poisson (λ) random variables.

   a. (10%) Find the maximum likelihood (ML) estimator, and an asymptotic normal distribution for the estimator of $\exp\{-\lambda\}$.

   b. Suppose that, rather than observing the random variables in (a) precisely, only the events

$$X_i = 0 \text{ or } X_i > 0,$$

   for $i = 1, \ldots, n$ are observed.

      I. (10%) Find the ML estimator of λ under this new observation scheme.

      II. (10%) In this new scheme, when does the ML estimator not exist (at a finite value in the parameter space) ? Justify your answer.