國立成功大學
114學年度碩士班招生考試試題

編　號：166

系　所：統計學系

科　目：數理統計

日　期：0211

節　次：第 2 節

注　意：1.不可使用計算機
　　　　2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. (15%, 5% for each) Consider the experiment of tossing two fair dice. Define

$$X = \text{sum of the two dice} \quad \text{and} \quad Y = |\text{difference of the two dice}|.$$

(a) Let $A = \{(x,y): X = 7 \text{ and } Y \leq 4\}$. Find $P((X,Y) \in A)$.

(b) Compute the expectation of $3X + 2Y$, that is, $E(3X + 2Y)$.

(c) Compute the expectation of XY, that is, $E(XY)$.

2. (20%, 10% for each) Let $X_1, X_2, \ldots, X_n$ be an independent and identically distributed (iid) sample from $\text{Poisson}(\lambda)$, where λ is an unknown parameter.

(a) Find a level α likelihood ratio test (LRT) for $H_0: \lambda = \lambda_0$ versus $H_1: \lambda \neq \lambda_0$.

(b) Assume that a prior $\lambda \sim \text{Gamma}(a, b)$. Construct a $100(1-\alpha)\%$ credible set for λ.

3. For $W = \{(X_i, Y_i): i = 1, \ldots, n\}$. Consider the model $Y_i = \beta X_i + \epsilon_i$, for $i = 1, \ldots, n$, where $\epsilon_1, \ldots, \epsilon_n$ are independent and identically distributed (iid) $\mathcal{N}(0, \sigma^2)$.

(a) (10%) Assume that $X_1, \ldots, X_n$ are fixed constants, find the maximum likelihood estimators (MLE) for β and σ^2.

(b) (5%) Find the distribution of the MLE of β.

(c) (10%) If, now, assume that $X_1, \ldots, X_n$ are iid $\mathcal{N}(\mu_x, \sigma_x^2)$ random variables and independent of $\epsilon_1, \ldots, \epsilon_n$. Find the MLE for β, σ^2, μ_x and σ_x^2.

4. (40%, 10% for each) Let $X \sim \mathcal{N}(\mu_1, \sigma^2)$ and $Y \sim \mathcal{N}(\mu_2, \sigma^2)$, where X and Y are independent.

(a) Define $L_1 = a_1 X + a_2 Y$ and $L_2 = a_3 X + a_4 Y$. Characterize the collection of all real constants a_1, a_2, a_3, a_4 such that the random variables L_1 and L_2 are independent.

(b) Are the random variables

$$V = \frac{(X+Y)}{\sqrt{2}\sigma} \quad \text{and} \quad W = \frac{(X-Y)}{\sqrt{2}\sigma}$$

independent? Why?

(c) Under the case $\mu_1 = \mu_2$, determine the marginal distributions of V and W.

(d) Under the case $\mu_1 = \mu_2$, obtain the conditional distribution of

$$P = \frac{XY}{\sigma^2} \quad \text{given that} \quad T = \frac{(X+Y)^2}{2\sigma^2} = t.$$