國立成功大學

115學年度碩士班招生考試試題

編　號：155

系　所：統計與資料科學學系

科　目：數學

日　期：0204

節　次：第 1 節

注　意：1.不可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

(20%) 1. True or False (Please answer "O" for True and "X" for False) (2% for each):

(1) All continuous functions have antiderivatives.

(2) If $f''(c) = 0$, then $(c, f(c))$ is the inflection point of curve $f(x)$.

(3) If $f'(x)$ exists and is nonzero for all x, then $f(1) \neq f(0)$.

(4) If both partial derivatives $f_x(a,b)$ and $f_y(a,b)$ exist, then f is continuous at (a,b).

(5) $\sum_{n=1}^{\infty} n^{-\sec n}$ is convergent.

(6) Any set containing the zero vector is linearly dependent.

(7) If A is an orthogonal matrix, then A^n is also an orthogonal matrix, $n \in \mathbb{N}$.

(8) For any non-zero square matrix, AA^T and A^TA are orthogonally diagonalizable.

(9) $\mathbb{R}^n$ is a subspace of $\mathbb{R}^{n+1}$.

(10) If A is square matrix and $Ax = \lambda x$ for any λ, then x is the eigenvector of A.

(15%) 2. Evaluate the followings:

(a)(5%) $\lim_{x\to\infty} \left[x - x^2 \ln\left(\frac{x+1}{x}\right)\right]$=?

(b)(5%) $\int_1^4 \frac{1}{2x(x+\sqrt{x})}\, dx$=?

(c)(5%) $\int \frac{x^2}{\sqrt{1-x^2}}\, dx$=?

(15%) 3. Let the sequence $\{a_n\}_{n\geq 1}$ be

$$a_1 = 2 \text{ and } a_n = \frac{1}{3 - a_{n-1}}, \forall n \geq 2.$$

(a)(10%) Show that $\{a_n\}_{n\geq 1}$ is convergent. (Hint: Monotonic Sequence Theorem)

(b)(5%) Find $\lim_{n\to\infty} a_n$=?

(10%) 4. Evaluate

$$\iiint_E x dV = ?$$

where E is the solid tetrahedron with vertices (0,0,0), ($\frac{1}{3}$,0,0), (0,1,0) and (0,0,1).

(10%) 5. Show that if A and B are any $n \times n$ matrices, then det(AB)=det(A)det(B).

(15%) 6. Let $B_1 = \{f_1, f_2\}$ be a basis for vector space V with $f_1 = \sin x$ and $f_2 = \cos x$.

(a)(5%) Show that $B_2 = \{g_1, g_2\}$ is also a basis for V with $g_1 = 2\sin x + \cos x$, $g_2 = 3\cos x$.

(b)(5%) Compute the transition matrix from B_1 to B_2. That is, $P_{B_1 \to B_2}$=?

(c)(5%) Compute $[w]_{B_2}$ by using $P_{B_1 \to B_2}$ for $w = 2\sin x - 5\cos x$.

(15%) 7. Suppose that a conic is given by the equation

$$5x^2 + 4xy + 5y^2 = 9$$

(a)(10%) Rotate x- and y-axes to the place the conic in standard position. Find the equation of the conic after the rotation.

(b)(5%) Determine the angle of rotation used in part (a).