國立成功大學

115學年度碩士班招生考試試題

編　號：156

系　所：統計與資料科學學系

科　目：數理統計

日　期：0204

節　次：第 2 節

注　意：1.不可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. (15%) Let $X_1, \ldots, X_n$ be a random sample from a population with pdf

$$f(x|\theta) = \frac{1}{2\theta}, \qquad -\theta < x < \theta,\ \theta > 0.$$

Find, if one exists, an uniformly minimum variance unbiased estimator of θ.

2. (15%) Find $P(X < \sqrt{Y})$ if X and Y are jointly distributed with pdf

$$f(x, y) = e^{-y}, \qquad 0 < x < y < \infty.$$

3. (15%, 5% for each) Let $X_1, \ldots, X_n$ be a iid $U(0, \theta)$.

(a) Find the Cramer-Rao lower bound (CRLB) for any unbiased estimator.

(b) Find the unbiased estimator based on the maximum likelihood estimator.

(c) Evaluate the variance of (b). Does this variance attain the CRLB? Explain the reason.

4. (15%) For the three-stage hierarchical model

$$Y|N \sim \text{Bin}(N, p) \qquad \text{and} \qquad N|\Lambda \sim Poisson(\Lambda) \qquad \text{and} \qquad \Lambda \sim Gamma(\alpha, \beta).$$

Evaluate $Var(Y)$ and $Cov(Y, \Lambda)$.

5. (10%) Let X has pdf $f_X(x) = c(x + 1)$, $-2 \leq x \leq 1$. Find the value of c that makes f a pdf. Then, find the pdf of $Y = X^2$.

6. (15%) Let $X_1, \ldots, X_n$ be a random sample from a $\text{N}(\theta, \sigma^2)$ population. Consider testing

$$H_0 : \theta_1 \leq \theta \leq \theta_2, \qquad \text{versus} \qquad H_1 : \theta \leq \theta_1 \text{ or } \theta \geq \theta_2.$$

Show that the test

$$\text{reject } H_0 \text{ if } \bar{X} > \theta_2 + t_{n-1,\alpha/2}\sqrt{S^2/n} \text{ or } \bar{X} < \theta_1 - t_{n-1,\alpha/2}\sqrt{S^2/n}$$

is not a size α test.

7. (15%) Find a $1 - \alpha$ confidence interval for θ, given $X_1, \ldots, X_n$ iid with pdf $f(x|\theta) = 2x/\theta^2$, $0 < x < \theta$.