第一部份：微積分（50分，每題10分）

注意：請寫詳細演算過程，否則扣分或不予計分.

1. 請利用極限定義証明 $\lim_{x\to 4}(3x-5)=7$.

   註：極限定義為 $\forall \varepsilon>0$, there is a $\delta>0$ such that

   if $0<|x-a|<\delta$, then $|f(x)-L|<\varepsilon$.

2. 利用剝殼法（或圓柱殼法或 Cylindrical shells）求由 $y=x^2$ and $y=x+2$ 所圍面積旋轉 $x=3$ 一圈所構成之體積.

3. 求下列之積分

   ⓐ $\int \frac{(1-x^2)^{3/2}}{x^6}\,dx$　　ⓑ $\int \frac{1}{(x+1)\sqrt{x-2}}\,dx$

4. 設 $f(x,y)=xy-x^2+1$，而 S 之範圍為 $x-2y+2=0$，$x+3y-3=0$，$y=0$ 所定，試求 $f(x,y)$ 在 S 上之二重積分

5. A revolving beacon（灯塔）in a lighthouse makes one revolution every 15 seconds. The beacon is 200 feet from the nearest point P on a straight shoreline（海岸線）. Find the rate at which a ray from the light moves along the shore at a point 400 feet from P.
   Note: The light revolves four times per minute.

第二部份：線性代數（50分，每題10分）

注意：請寫詳細演算過程，否則扣分或不予計分.

6. Let I be the interval $[-1,1]$; find an orthogonal basis for the vector space of polynomials on I of the form $a+bx+cx^2$, using the usual inner product on I

7. Using the Normalized Gram-Schmidt process, find an orthonormal basis for the subspace V of $\mathbb{R}^4$ spanned by the vectors

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$(1, 1, -1, -1)$, $(3, -1, 1, -3)$, $(1, 1, -3, 1)$

8. Prove that every eigenvalues of a $k \times k$ positive definite matrix A is positive.

9. 試証 $W = \{(a_1, \cdots, a_n) \in F^n : a_1 + \cdots + a_n = 0\}$ 為 F^n 之一子空間.

10. $V = \left\{ \begin{pmatrix} a & a+b \\ 0 & c \end{pmatrix} : a, b, c \in F \right\}$, 試找一 isomorphism 從 V 映至 F^3.

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