註1:前五題為微積分;後五題為線性代數;共計十題,每題10分.

註2:請寫詳細演算過程,否則扣分或不予計分.

1. Show that $f(x) = \sqrt{2x+1} + 2x$ is continuous at 3.

2. ⓐ 求積分 $\int_0^\pi \frac{1-\sin x}{x+\cos x}\, dx$

   ⓑ 求序列 (Sequence) 收斂或發散;若收斂,求其極限 (limit): $\{\sqrt{n+1} - \sqrt{n}\}$

3. Let $\mathbb{R}$ be the region bounded by the graphs of $x = \sqrt{8-y}$, $y = 2x$, $x+y+4=0$, and let f be continuous on $\mathbb{R}$. Express $\iint_{\mathbb{R}} f(x,y)\, dA$ as a sum of two iterated integrals of (a) Type I (dy dx) and (b) Type II (dx dy).

4. 當太空梭發射進入太空時,太空人之体重遞減至無重量狀態. 一個 150 磅的太空人在海平面高度 x 公里 (km) 時之体重 $W = 150\left(\frac{6400}{6400+x}\right)^2$. 若太空梭沿著地球大氣層移動率是 6 km/sec . 當 x = 1000 km 時, W 之遞減率為何?

5. Sketch the graph of a continuous function f. If n is an odd integer, then $f(n)=1$ and $f'(n)=0$; if n is an even integer, then $f(n)=0$ and $f'(n)$ does not exist; if n is any integer, then

   (a) $f'(x) > 0$ whenever $2n < x < 2n+1$

   (b) $f'(x) < 0$ whenever $2n-1 < x < 2n$

   (c) $f''(x) < 0$ whenever $2n < x < 2n+2$

6. 証明:若 A 和 B 為同階之可逆方陣,則 $(A \times B)^{-1} = B^{-1} \times A^{-1}$.

7. 証明 $A = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ 為正交矩陣.

8. Find a basis for the orthogonal complement of the subspace spanned by $(1,1,0,2)$, $(-1,2,1,0)$, $(0,3,1,2)$.

9. 試証 $\{u, v, w\}$ 為線性獨立, 若且唯若 $\{u+v, u+w, v+w\}$ 為線性獨立.

10. A certain population of animals has two age groups, "young" and "old". Suppose there are initially 200 young animals and 100 old ones, that the birthrates for young animals is $\frac{1}{3}$ and for old ones is 4, and that the probability a young animals will survive to become old is $\frac{1}{6}$. Determine the number of animals in each age group as time goes to ∞.